New Insights from One-Dimensional Spin Glasses

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Abstract. The concept of replica symmetry breaking found in the solution of the mean-field Sherrington-Kirkpatrick spin-glass model has been applied to a variety of problems in science ranging from biological to computational and even financial analysis. Thus it is of paramount importance to understand which predictions of the mean-field solution apply to non-mean-field systems, such as realistic short-range spin-glass models. The one-dimensional spin glass with random power-law interactions promises to be an ideal test-bed to answer this question: Not only can large system sizes—which are usually a shortcoming in simulations of high-dimensional short-range system—be studied, by tuning the power-law exponent of the interactions the universality class of the model can be continuously tuned from the mean-field to the short-range universality class. We present details of the model, as well as recent applications to some questions of the physics of spin glasses. First, we study the existence of a spin-glass state in an external field. In addition, we discuss the existence of ultrametricity in short-range spin glasses. Finally, because the range of interactions can be changed, the model is a formidable test-bed for optimization algorithms.

1 Introduction

Spin glasses [1–3] are paradigmatic models which can be applied to a wide variety of problems and fields ranging from economical to biological, as well as sociological problems, to name a few. Most prominent is the replica symmetry breaking solution of Parisi [4] of the mean-field Sherrington-Kirkpatrick (SK) spin glass. Unfortunately, an analytical solution for short-range realistic spin-glass models, such as the Edwards-Anderson Ising spin glass [5], remain to be found and generally phenomenological descriptions, such as the droplet picture [6] or numerical simulations are used to understand these systems. Given the lack of rigorous results for short-range spin glasses, it is of importance to understand the applicability of different predictions made by the mean-field solution of the SK model, as well as other theoretical pictures.

Unfortunately, numerical studies of spin glasses are difficult to accomplish and in general only small to moderate system sizes can be accessed. Despite huge technological advances in the last decade which have enabled the construction of large computer clusters out of commodity components, brute force computation alone will not suffice to probe considerably larger system sizes. The source of this problem lies in the diverging equilibration times of Monte Carlo simulations of spin glasses; the systems are generally NP hard. Furthermore, to obtain thermodynamically sound results, calculations need to be disorder averaged, thus adding considerable overheard to any simulation. To properly probe the thermodynamic limiting behavior it is thus important to use efficient algorithms, improved models, as well as large computer clusters.

We discuss in detail a one-dimensional spin-glass model with power-law interactions [6–8] where, by tuning the exponent of the power-law interactions different universality classes from infinite-range SK to short-range can be probed. Furthermore, because the model is one-dimensional, a wide range of system sizes can be probed. In the past we have applied the model to different problems in the physics of spin glasses [8–13]. In this work we study two questions which lie at the core of the applicability of the mean-field solution to short-range spin glasses: Do short-range spin glasses order in an externally-applied magnetic field? Are short-range spin glasses ultrametric? Our results suggest that new theoretical descriptions are needed: While there are indications of an ultrametric structure of phase space, spin-glass order is destroyed in a field for short-range systems.

Finally, we also discuss extensions as well modifications of the model to study different related problems in the physics of spin glasses and present applications to algorithm development and testing.

2 Model & Numerical Method

We first introduce the one-dimensional Ising chain in detail and explain its rich phase diagram. Furthermore, we describe exchange (parallel tempering) Monte Carlo, a numerical method which is very efficient to study spin-glass systems at low temperatures.

2.1 The one-dimensional Ising chain

The Hamiltonian of the one-dimensional Ising chain with power-law interactions [7,14,8] is given by

$$\mathcal{H}_{\rm 1D} = -\sum_{i<j} J_{ij} S_i S_j - \sum_i h_i S_i \, , \qquad J_{ij} = c(\sigma)\frac{\epsilon_{ij}}{{r_{ij}}^{\sigma}} \, , \tag{1}$$

where $S_i \in \{\pm 1\}$ are Ising spins and the sum ranges over all spins in the system. To ensure periodic boundary conditions, the L spins are placed on a ring, see Fig. 1 (right panel). Here, $r_{ij} = (L/\pi)\sin(\pi|i-j|/L)$ is the distance between the spins on the ring topology and ϵ_{ij} are Gaussian-distributed random couplings of zero mean and standard deviation unity. The constant $c(\sigma)$ is chosen such that the mean-field transition temperature to a spin-glass

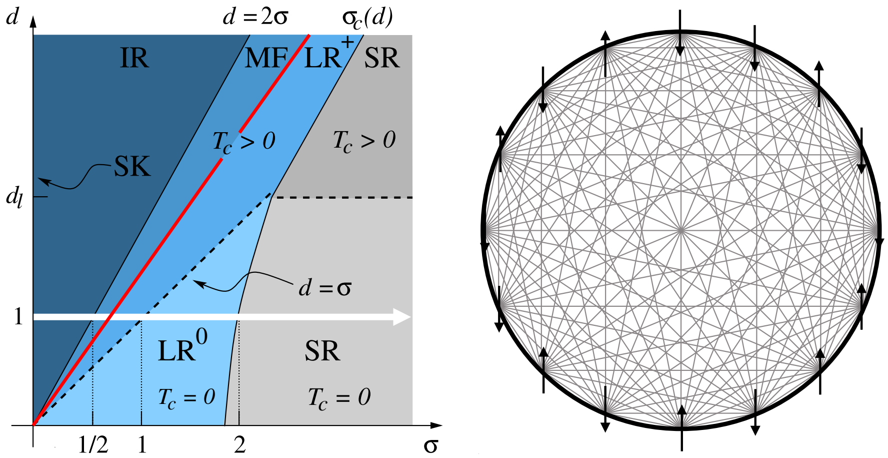

Fig. 1. Left panel: Schematic phase diagram of the one-dimensional Ising chain with power-law interactions [14]. The white horizontal arrow corresponds to $d = 1$. For $\sigma \leq 1/2$ we expect infinite-range (IR) behaviour reminiscent of the SK model. For $1/2 < \sigma \leq 2/3$ we have mean-field (MF) behaviour corresponding to an effective space dimension $d_{\mathrm{eff}} \geq 6$, whereas for $2/3 < \sigma \lesssim 1$ we have a long-range (LR^+) spin glass with a finite ordering temperature T_{c}. In these regimes $d_{\mathrm{eff}} \approx 2/(2\sigma - 1)$ [1]. For $1 \leq \sigma \lesssim 2$ we have a long-range spin glass with $T_{\mathrm{c}} = 0$ (LR^0) and for $\sigma \gtrsim 2$ the model displays short-range (SR) behaviour with $T_{\mathrm{c}} = 0$. Figure adapted from Ref. [8]. Right panel: Graphical representation of the one-dimensional Ising chain with $L = 16$ spins.

phase is $T_c^{\mathrm{MF}} = 1$; see Ref. [8] for details. The model has a very rich phase diagram in the d–σ plane, see Fig. 1 (left panel). In this work we are interested in $d = 1$ which corresponds to the thick horizontal white arrow in the phase diagram. The universality class and range of the interactions of the model can be continuously tuned by changing the power-law exponent σ. Furthermore, there are theoretical predictions for the critical exponents [7]: $\nu = 1/(2\sigma - 1)$ for $\sigma \leq 2/3$, and $\eta = 3 - 2\sigma$. Therefore, predictions made for the mean-field spin-glass can be probed when the effective space dimension (range of the interactions) is reduced. Furthermore, because the efficiency of different algorithms often depends strongly on the range of the interactions, the one-dimensional chain is an ideal test bed to benchmark the efficiency of optimization algorithms.

In one space dimension, for $\sigma \leq 1/2$ the model is in the Sherrington-Kirkpatrick [15] infinite-range universality class where the energy of the system needs to be rescaled with the system size to avoid divergencies. In particular, for $\sigma = 0$ the SK model is recovered exactly. For $1/2 < \sigma < 1$ the model has a finite-temperature spin-glass ordering transition. Furthermore, for $1/2 < \sigma \leq 2/3$ the system is in the mean-field universality class corresponding to a high-dimensional short-range spin-glass system above the upper critical dimension $d_{\mathrm{u}} = 6$. For $2/3 < \sigma < 1$ the system is non-mean field, whereas for $\sigma \geq 1$ the spin-glass phase only exists at $T = 0$, i.e., the lower critical dimension of short-range spin glasses corresponds to $\sigma = 1$.

2.2 Numerical method

Because of a rough energy landscape and diverging relaxation times, spin glasses are extremely difficult to study numerically. Any numerical method used must have the potential to efficiently cross energy barriers and thus sample the phase space evenly. Probably one of the simplest, yet most efficient methods to study problems with rough energy landscapes (beyond spin glasses) is the exchange (parallel tempering) Monte Carlo method [16].

The idea behind the method is to allow for a Markov process in temperature space. M copies of the system are simulated at different temperatures, where the largest temperature is generally chosen to be of the order of $2T_c^{\mathrm{MF}}$. Besides the simple Monte Carlo updates [17] on each spin of the system, after a certain number of lattice sweeps the energies of neighboring temperatures are compared and a Monte Carlo move which swaps the temperatures of neighboring configurations is proposed. With this approach, a configuration stuck in a metastable state has the possibility to heat up and then cool back down to the true equilibrium state thus effectively speeding up equilibration by orders of magnitude. The position of the temperatures has to be chosen with care: If neighboring temperatures are chosen too far apart, a bottleneck in the temperature-space Markov process emerges thus reducing the efficiency of the method. If the temperatures are too close extra unnecessary overhead is introduced. To select the position of the temperatures, it is convenient to study the acceptance probabilities of the global Monte Carlo moves. Because in spin glasses the susceptibility does not diverge, a generally good thumb-rule is to select the position of the temperatures such that the probabilities are between 0.2 and 0.9 and roughly independent of temperature. This is not necessarily the case for other systems. We also refer the reader to Ref. [18] where an iterative feedback method is introduced which ensures that the random walk of each configuration in temperature space is optimal.

When using Gaussian-distributed disorder we test equilibration of the Monte Carlo simulations by equating the link overlap q_l to the energy $U = -(1/N)\sum_{i,j}[J_{ij}\langle S_i S_j\rangle]_{\mathrm{av}}$ of the system [19], i.e.,

$$U(q_l) = \frac{(T_c^{\mathrm{MF}})^2}{2T}(q_l - 1) \, , \qquad q_l = \frac{2}{N}\sum_{i,j}\frac{[J_{ij}^2]_{\mathrm{av}}}{(T_c^{\mathrm{MF}})^2}[\langle S_i S_j\rangle_T^2]_{\mathrm{av}} \, . \quad (2)$$

In Eq. (2), $\langle\cdots\rangle_T$ represents a thermal average and $[\cdots]_{\mathrm{av}}$ an average over the disorder. T is the temperature of the system. As can be seen in Fig. 2, starting from a random configuration will underestimate $U(q_l)$, whereas the energy U will be overestimated. Once both agree, the system is in thermal

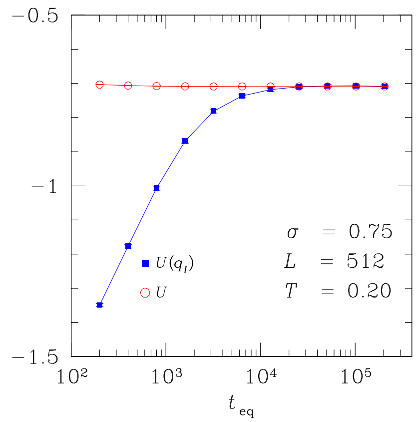

Fig. 2. Equilibration test using Eq. (2). Once the energy U computed directly and from the link overlap $[U(q_l)]$ agree, the system is in thermal equilibrium. Data for $L = 512$, $T = 0.20$ and $\sigma = 0.75$.

equilibrium. Note that the method can be easily extended to system with (Gaussian distributed) external fields [11].

3 Selected results

We have applied the one-dimensional Ising chain to several problems in the physics of spin glasses. Below we present in more detail two questions which lie at the core of the applicability of the mean-field solution to short-range spin glasses. In the following we compare the mean-field SK model ($\sigma = 0$) to the one-dimensional Ising chain for $\sigma = 0.75$ where the model is in the non-mean-field universality class.

3.1 Do spin glasses order in a magnetic field?

The applicability of spin-glass models to other fields of science relies heavily on the existence of a spin-glass phase in a field. Many mappings onto spin-glass models produce external field terms. While the mean-field model has been shown to have a spin-glass phase in a field, it has been unclear until recently if short-range spin glasses order in a field as well [20–29]. Simulations of three-dimensional spin-glass models [28,30] suggest that the de Almeida-Thouless line [31], which separates the spin-glass from the paramagnetic state in the H–T phase diagram does not exist for realistic short-range Ising spin glasses. Although the aforementioned studies in three space dimensions using the finite-size two-point correlation length [32] provide clear evidence that short-range spin glasses do not order in a field, they do not shed any light on the behavior of short-range spin glasses with space dimensions above the upper critical dimension.

In Ref. [11] the one-dimensional Ising chain has been studied in an externally applied Gaussian-distributed random field—which has a similar behavior than a uniform field —for different exponents σ of the power-law interactions. For exponents which correspond to effective space dimensions above the upper critical dimension, a spin-glass state in a field is found, whereas for exponents $\sigma > 2/3$ which correspond to effective space dimensions less than six, no de Almeida-Thouless line could be found for simulations down to very low temperatures. Technical details about the simulation, and in particular the parameters of the simulation can be found in Ref. [11].

In order to probe the existence of a spin-glass state we add an external (random) field to the Hamiltonian, i.e., $\mathcal{H}_{\rm 1D} \to \mathcal{H}_{\rm 1D} - \sum_i h_i S_i$. The reasons for using random fields are the ability to thoroughly test for equilibration of the Monte Carlo method (for detail see Refs. [28] and [11]). Furthermore, exchange Monte Carlo performs better.

To test for the existence of the transition for $\sigma > 1/2$, we compute the finite-size correlation length from the Fourier transform of the spin-glass susceptibility [33,32]:

$$\chi_{\rm SG}(k) = \frac{1}{N} \sum_{ij} \left[\left(\langle S_i S_j \rangle_T - \langle S_i \rangle_T \langle S_j \rangle_T \right)^2 \right]_{\rm av} e^{ik(R_i - R_j)} \, . \tag{3}$$

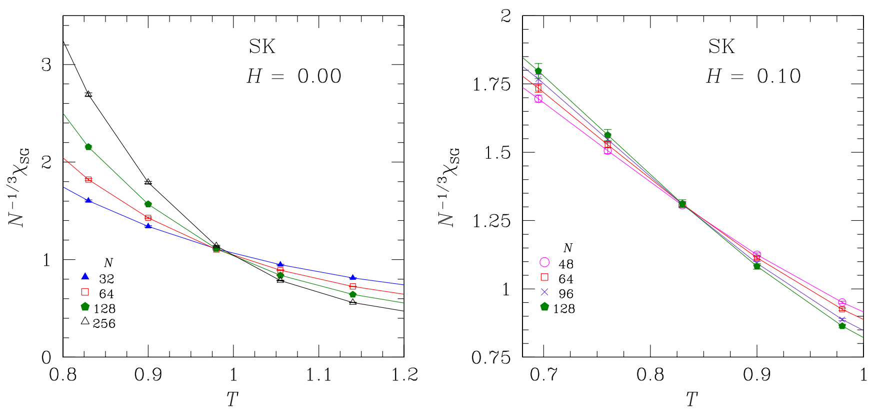

Fig. 3. Left panel: Scaled spin-glass susceptibility $N^{-1/3}\chi_{\rm SG}$ as a function of temperature for the mean-field Sherrington-Kirkpatrick model at zero external field. The data cross at $T_c(H = 0) = 1.0$ in agreement with analytical results. Right panel: Same observable and model as depicted in the left panel, except for $H = 0.10$. The data cross at $T_c(H = 0.10) \approx 0.82$ thus clearly showing that the mean-field model orders in a field, as expected from theoretical results.

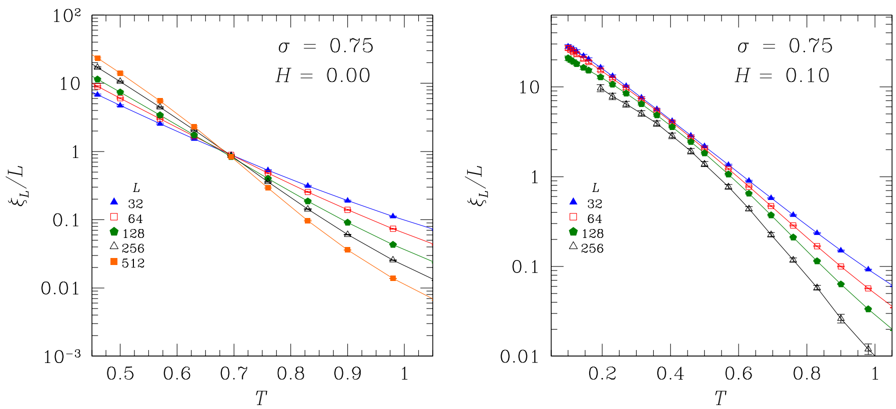

Fig. 4. Left panel: Finite-size correlation length divided by the system size as a function of temperature for the one-dimensional Ising spin chain with $\sigma = 0.75$ at zero field. In this regime the system is not in the mean-field universality class. The data cross cleanly at $T_c(H = 0) \approx 0.69$. Right panel: Same observable and model as the left panel, except for $H = 0.10$. Note that for temperatures as low as $T = 0.1$ there is no crossing visible, suggesting that there is no spin-glass state in a field. Figure adapted from Ref. [11].

After performing an Ornstein-Zernicke approximation we obtain for the two-point finite-size correlation length

$$\xi_L = \frac{1}{2\sin(k_{\rm min}/2)} \left[\frac{\chi_{\rm SG}(0)}{\chi_{\rm SG}(k_{\rm min})} - 1\right]^{1/(2\sigma-1)} , \tag{4}$$

where $\chi_{\rm SG}(0)$ is the standard spin-glass susceptibility and $k_{\rm min} = 2\pi/L$. The finite-size correlation length divided by the system size is a dimensionless quantity which scales as $\xi_L/L = \tilde{X}[L^{1/\nu}(T - T_c)]$. Because in the infinite-range universality class no correlation length can be computed, we exploit the fact that the critical exponent $\eta = 1/3$ is exactly known for the SK model [34,30]. Therefore, we locate the transition in the SK model by studying $\chi_{\rm SG}/N^{1/3} = \tilde{C}[L^{1/\nu}(T - T_c)]$, where $\chi_{\rm SG} = \chi_{\rm SG}(k = 0)$. Once the respective observables for different system sizes cross we have a spin-glass state for $T \le T_c$, where T_c is given by the crossing point.

In Fig. 3 we show $\chi_{\rm SG}/N^{1/3}$ for the SK model ($\sigma = 0$) as a function of temperature for zero, as well as an external field of strength $H = 0.10$. In both cases the data cross, indicative of a transition in zero as well as finite fields. This is not the case for the one-dimensional model with $\sigma = 0.75$. While the data of the finite-size correlation length at zero field clearly show a transition at $T_c \approx 0.69$ (see Fig. 4, left panel), this is not the case for

$H = 0.10$ where the data do not cross even for temperatures considerably lower than the critical temperature (see Fig. 4, right panel).

The presented results clearly show the numerical existence of an AT line for the mean-field SK model, whereas for the model at $\sigma = 0.75$ (outside the mean-field universality class there is no sign of a transition in a small but finite field). Together with results presented in Ref. [11] we thus conclude that short-range spin glasses below the upper critical dimension do not order in an externally-applied magnetic field.

3.2 Are spin glasses ultrametric?

Ultrametricity is an intrinsic property of the Parisi solution of the mean-field model [35] and it can be described in the following way: Consider an equilibrium ensemble of states at $T < T_{\rm c}$ and pick three, S^α, S^β and S^γ, at random. Order them so that their distances $d_{\alpha\beta} = (1 - q_{\alpha\beta})/2$, where $q_{\alpha\beta} = L^{-1}\sum S_i^\alpha S_i^\beta$ is the spin overlap, satisfy $d_{\alpha\gamma} \geq d_{\gamma\beta} \geq d_{\alpha\beta}$. Ultrametricity means that in the thermodynamic limit we obtain $d_{\gamma\beta} = d_{\alpha\beta}$ with probability 1, i.e., the states lie on an isosceles triangle.

To date, the existence of ultrametricity for short-range spin glasses—which would validate the applicability of the mean-field solution to short-range systems—is highly controversial. Recent results [36] suggest that short-range systems are not ultrametric, whereas other opinions exist [37–39]. Because the one-dimensional Ising chain allows for tuning the system away from the mean-field universality class, it presents itself as the ideal test-bed for this problem. Below we present results for $\sigma = 0.0$ (SK) as well as 0.75 (non-mean-field regime) using an approach closely related to the one used by Hed *et al.* [36].

We generate 1000 equilibrium states (spin configurations) for $1000 - 4000$ disorder instances of the model using exchange Monte Carlo at $T = 0.4T_c$ (i.e., $T = 0.4$ for the SK model and $T = 0.27$ for the one-dimensional chain with $\sigma = 0.75$). The temperature used is chosen such that we probe deep in the spin-glass phase, but not too low to avoid trivial state triangles. The generated states are in turn sorted using Ward's hierarchical clustering approach [40] (see Fig. 5). The clustering procedure starts with L clusters which contain one state and the two closest lying clusters are merged. Distances are measured in terms of the hamming distance $d_{\alpha\beta} = (1 - q_{\alpha\beta})/2$. This procedure is repeated until one large cluster is obtained. Once the states are clustered, we select three states from different branches of the left sub-tree (see Ref. [36] for details) and sort the distances: $d_{\rm max} \geq d_{\rm med} \geq d_{\rm min}$. We compute the correlator

$$K = \frac{d_{\rm max} - d_{\rm med}}{\varrho(d)}, \qquad (5)$$

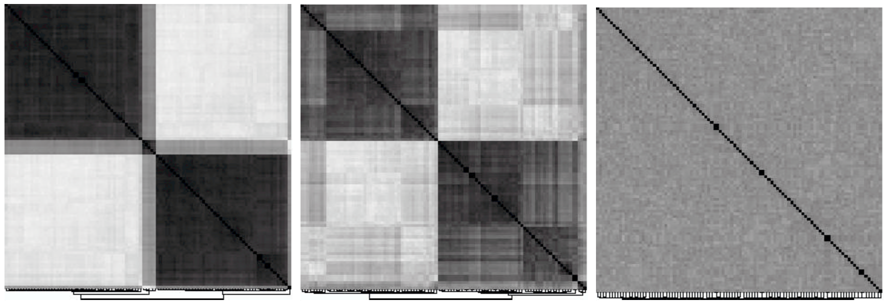

Fig. 5. Dendrograms and distance matrices. Darker colors correspond to closer distances in phase space. Left panel: SK model at $T = 0.4$ ($L = 1024$). The distance matrix shows clear structure below T_c. Middle panel: One-dimensional Ising chain for $\sigma = 0.75$ and $T = 0.40 < T_c$ ($L = 512$). Again the data show structure. This is in contrast to the right panel which shows data for the one-dimensional chain at $T = 1.40 \gg T_c$ ($L = 512$, $\sigma = 0.75$).

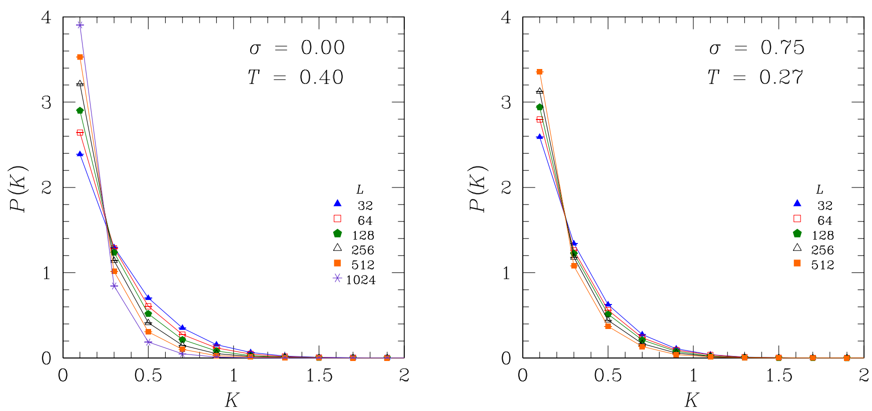

Fig. 6. Left panel: Distribution $P(K)$ for the mean-field SK model at $T = 0.4T_c$. The data peak for $K \to 0$ with increasing system size showing clearly that phase space is ultrametric. Right panel: Same observable as in the left panel for the one-dimensional Ising chain with $\sigma = 0.75$ (non-mean-field universality class) at $T = 0.27 \approx 0.4T_c$. While the divergence at $K = 0$ is less pronounced, the data show a similar behavior than in the left panel.

where $\varrho(d)$ is the width of the distribution of distances. If the space is ultrametric, we expect $d_{\rm max} = d_{\rm med}$ for $L \to \infty$. This means for the distribution $P(K) \to \delta(K = 0)$ for $L \to \infty$.

In Fig. 6 (left panel) we show data for the distribution $P(K)$ for the SK model at $T = 0.4T_c$. For increasing system size the data seem to converge to a limiting delta function. This is not the case for $T = T_c$ (not shown) where the data are independent of system size and show no divergence for $K \to 0$. This suggests that the used observable correctly captures the underlying ultrametric behavior. Furthermore, studies of cophonetic distances show that the structures found in the dendrograms are not arbitrary. Figure 6 (right panel) shows $P(K)$ for the one-dimensional Ising chain with $\sigma = 0.75$ at $T = 0.27 \approx 0.4T_c$ for a range of system sizes. The data show a similar behavior than for the SK model, although the effect is not as pronounced. Further simulations at σ values larger than 0.75 as well as a quantitative study of the number of clusters and RSB layers shall clarify with certainty if short-range spin glasses have an ultrametric phase structure or not.

4 Future directions

In the past, we have studied several properties of spin glasses using the one-dimensional Ising chain, such as the nature of the spin-glass state [8,9,41], ground-state energy distributions of spin glasses [10], the existence of a spin-glass state in a field [11] (see above), field chaos in spin glasses [13], local-field distributions in spin glasses [12], as well as ultrametricity in spin glasses [13] (see above). Furthermore, other groups have also studied other open questions in the physics of spin glasses with this model, such as nonequilibrium problems [42] or different cumulants of the order parameter distribution [43]. All previous studies had been done on the model presented in Eq. (1) using Ising spins. In this section we mention some extensions, as well as modifications of the model which can be used to study different problems.

4.1 Variations on the model

Recently, a one-dimensional spin-glass chain with Heisenberg spins has been studied in Ref. [44] to test the controversial spin-chirality decoupling scenario [45–48] proposed by Kawamura. It is unclear to date what the nature of the spin-glass state in Heisenberg spin glasses is. In particular, it is unclear if spin and chirality degrees of freedom decouple. To test this scenario, simulations of the one-dimensional Heisenberg chain [44] at $\sigma = 1.1$ have been performed. For $\sigma = 1.1$ the spin degrees of freedom only order at $T = 0$, whereas results suggest that the chirality degrees of freedom order at finite nonzero temperatures. Similar studies could be performed for models with planar XY spin degrees of freedom, as well as Potts spins (work in progress).

Finally, the Hamiltonian can also be modified to include, for example, p-spin interactions to study structural glasses [49] (work in progress). Preliminary results suggest that the model has a finite ordering temperature in the mean-field regime.

4.2 Studying larger systems with dilution

While the linear system sizes L studied with the one-dimensional Ising chain are considerably larger than the system sizes accessible in short-range systems, the fact that the model is fully-connected makes it difficult to access large numbers of spins because any algorithm would have to do $\mathcal{O}(L^2)$ updates at every Monte Carlo sweep. This is because the system has $\mathcal{O}(L^2)$ interactions between the spins. Recently, Leuzzi and collaborators suggested a variation of the model which is diluted, thus drastically reducing the number of neighbors for each spin [50]. In their version, a random bimodally-distributed bond between two spins is placed with a power-law dependent probability adjusted such that the mean connectivity z is always 6 for all σ. This has the effect that for $\sigma \to 0$ we recover the Viana-Bray model with fixed connectivity [51]. Because of the dilution, systems of 10^4 spins can be studied to temperatures as low as $\sim 0.4T_c$.

In Fig. 7 we present data for a diluted system with Gaussian-distributed random interactions and $\sigma = 0.75$. In this case, the probability to place a bond between two spins is $\mathcal{P}(J_{ij} \neq 0) = r^{-2\sigma}$, where r is the distance between the spins. The mean connectivity z of the model is then given by $z = 2\zeta(2\sigma)$ in the thermodynamic limit, where ζ is the Riemann zeta function. For $\sigma = 0$ we recover the SK model, whereas, for example, for $\sigma = 0.75$ the mean connectivity is only $z \approx 5.22$ thus allowing the study of large systems (note that the interactions are rescaled such that $T_c^{\rm MF} = 1$). In the left panel of

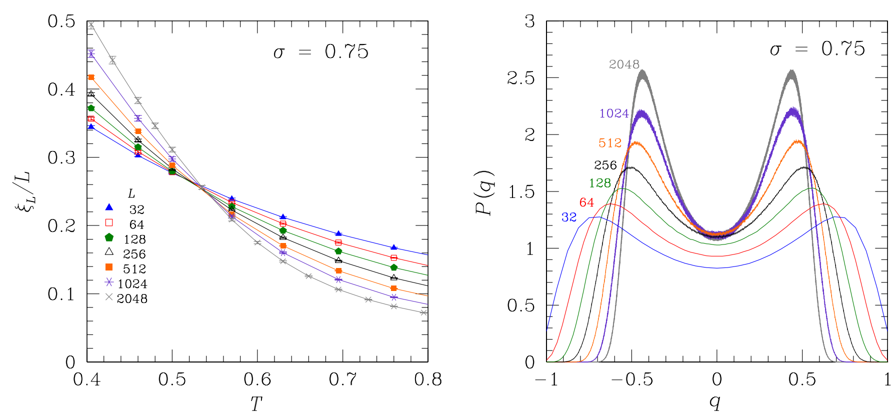

Fig. 7. Left panel: Finite-size correlation length for the power-law diluted one-dimensional Ising spin glass with variable connectivity. The data cross at $T_c \approx 0.54$ illustrating the existence of a transition. Right panel: Distribution of the spin overlap function $P(q)$ at $T = 0.4$ for different system sizes. The width of the lines corresponds to the error bars.

Fig. 7 the finite-size correlation length as a function of temperature is shown. The data cross at $T_c \approx 0.54$ signaling the existence of a spin-glass transition. In the right panel of Fig. 7 we show the distribution of the spin overlap $q = L^{-1} \sum_i S_i^{\alpha} S_i^{\beta}$. While the data show corrections due to critical fluctuations, they converge to a seemingly system-size independent value around $|q| \approx 0$. This would agree with the replica symmetry breaking scenario by Parisi [4,52,53,2] although lower temperatures are needed to properly address this question. Current work focuses on revisiting the existence of a spin-glass state in a field using the model with dilution.

5 Benchmarking of algorithms

Benchmarking optimization algorithms [55,56] plays a crucial role in the field of disordered and complex systems, as well as many other interdisciplinary applications. Knowing the range of applicability of a given algorithms can be of great importance when trying to solve a given problem. For example, whereas the branch, cut & price algorithm [57–59] works best for short-range systems, it is least efficient when the interactions are long range [10].

Recently, the hysteretic optimization heuristic [60] has been introduced to estimate ground states of spin-glass systems. The method is known to work well for the mean-field SK model, as well as the traveling salesman problem [56]. The idea behind hysteretic optimization is successive demagnetization at zero temperature. With additional shake-ups (field increases to further randomize the system) close-to-ground-state configurations can be obtained. Recently, Gonçalves and Bottcher [54] have studied the efficiency of the method on the one-dimensional Ising chain. Data adapted from their work shown in Fig. 8 clearly show that the method works best for infinite-

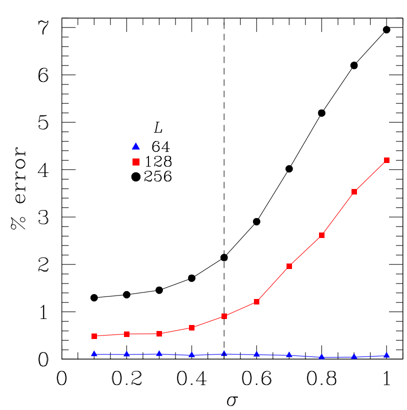

Fig. 8. Percentage error in the ground-state energies obtained with hysteretic optimization in comparison to exact ground states as a function of the exponent σ for different system sizes L. Clearly, the algorithm works best for $\sigma \lesssim 1/2$ (vertical dashed line), i.e., in the infinite-range regime. Outside the infinite-range regime, avalanches do not percolate the system and thus the algorithm is less efficient. Figure adapted from reference [54].

range models ($\sigma \leq 1/2$) where avalanches in the hysteresis loops proliferate easily. While the error in finding the ground states increases slightly with system size for $\sigma \lesssim 1/2$ the increase is considerably stronger for larger values of σ. As soon as the system is not infinite-ranged, avalanches are small and the method is not efficient.

6 Concluding remarks

By using a one-dimensional spin-glass model with random power-law interactions we have been able to shed some light on some of the open questions in the physics of spin glasses. The one-dimensional spin-glass chain has the advantage over conventional models that large linear system sizes can be studied. Furthermore, by changing the power-law exponent of the interactions, different universality classes ranging from the mean-field to the short-range universality class can be probed. The latter feature of the model allows also for efficient benchmarking of optimization algorithms.

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