Fractal dimension of domain walls in two-dimensional Ising spin glasses

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Outline

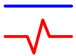
- Introduction
- Techniques
- Results
- Summary

Model

- $N = L \times L$ Ising spins $\sigma_i = \pm 1$ on square lattice
- Periodic boundary conditions in one direction
- Edwards-Anderson Hamiltonian: $\mathcal{H}(\sigma) = -\sum_{\langle ij \rangle} J_{ij}\sigma_i\sigma_j$

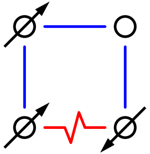
interaction strength:

$J_{ij} > 0$:
$J_{ij} < 0$:



quenched disorder

frustration:



- Always: global spin flip connects GS pairs, only:

$P(J_{ij}) \propto \exp\left(-J_{ij}^2/2\right)$

trivial GS-degeneracy

$P(J_{ij}) \propto [\delta(J_{ij}+1)+\delta(J_{ij}-1)]$

numerous degenerate GS

[A.K. Hartmann and H. Rieger, *Optimization Algorithms in Physics*]

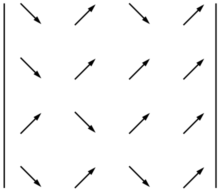
Domain Walls (DWs)

- Defined relative to 2 spin configurations $\sigma^{(1)/(2)}$
- $\sigma^{(1)}$:
  $\sigma^{(2)}$:
- Separates regions of agreeing/disagreeing spin config.

DW energy:

$$\Delta E = 2 \sum_{\langle ij \rangle \in \mathcal{D}} J_{ij} \sigma_i^{(1)} \sigma_j^{(1)}$$

$\mathcal{D} \equiv$ bonds satisfied by only 1 config.



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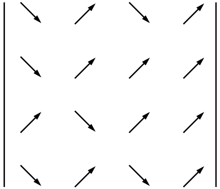

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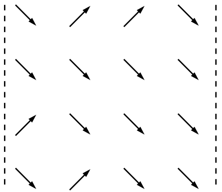

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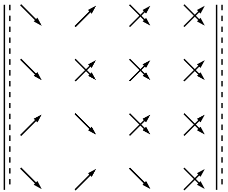

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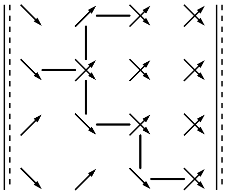

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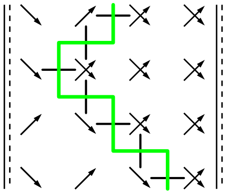
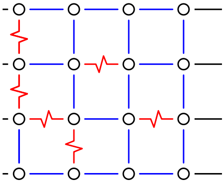


Dual Graph

- Construct weighted graph $G = (V, E, \omega)$
  $V(G)$ elementary plaquettes (EP)
  $E(G)$ connect EP with common side
  ω energy contribution to DW



Consider GS σ for periodic BCs:

(i) Bond satisfied for σ, e.g.

 : $\omega \geq 0$

(ii) Bond not satisfied for σ, e.g.

 : $\omega \leq 0$

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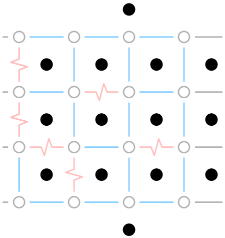

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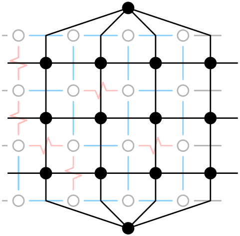

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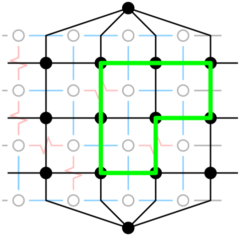

no loops with negative weight:

$$\omega(\mathcal{C}) = \sum_{\langle ij \rangle \in \mathcal{C}} J_{ij}\sigma_i\sigma_j \geq 0$$

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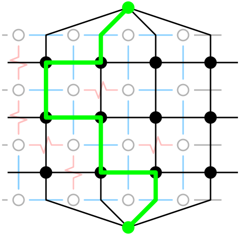


no loops with negative weight:

$$\omega(\mathcal{C}) = \sum_{\langle ij\rangle \in \mathcal{C}} J_{ij}\sigma_i\sigma_j \geq 0$$

- DW: minimum-weight $(\mathrm{top}, \mathrm{bottom})$ path

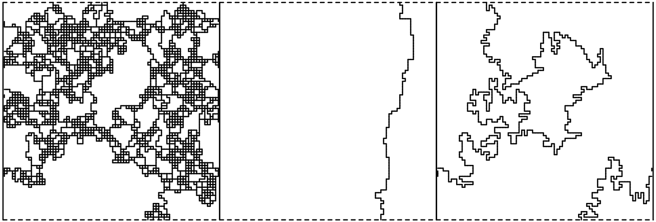
Minimum-Weight Paths

- G: undirected graph, allowing for negative edge weights
- Here: standard minimum-weight path algorithms, e.g. Bellman-Ford, Floyd-Warshall, don't work
- Minimum-weight path problem on dual requires matching techniques

  i) Dual graph $\rightarrow$ auxiliary graph
  ii) Find minimum-weighted perfect matching (MWPM)
  iii) Interpret MWPM as min.-weight path

[R.K. Ahuja, T.L. Magnanti and J.B. Orlin, *Network flows*]

Degeneracy

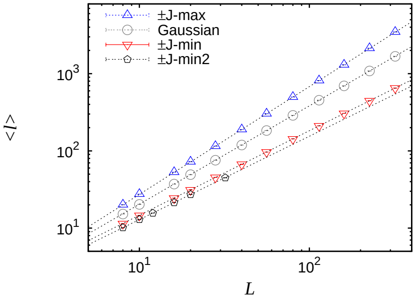


- $\pm J$ disorder $\to$ numerous DWs
- $\omega(e) \to \omega(e) + \epsilon$ minimal length DWs ($\pm J^{\mathrm{min}}$)
- $\omega(e) \to \omega(e) - \epsilon$ maximal length DWs ($\pm J^{\mathrm{max}}$, only lower bound)
- allow to change GS to yield true minimum length DWs ($\pm J^{\mathrm{min2}}$)

[OM and A.K. Hartmann, arXiv:0704.2004]

Fractal dimension of domain walls

- Scaling of DW length: $\langle \ell \rangle \sim L^{d_f}$, with $1 \leq d_f \leq 2$



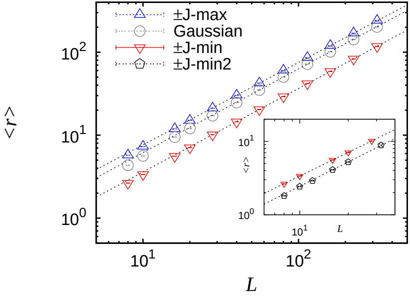
	d_f
Gaussian	1.274(1)
$\pm J^{\mathrm{min}}$	1.095(1)
$\pm J^{\mathrm{min2}}$	1.080(5)
$\pm J^{\mathrm{max}}$	1.395(1)

Gaussian: $d_f = 1.28(1)$
[D. Bernard *et al*, cond-mat/0611433]

- Gaussian: Conformal field-theory: relation $d_f - 1 = 3/[4(3+\theta)]$ between d_f and stiffness exp. $\Delta E \sim L^{\theta}$ in the context of stochastic Loewner evolution (SLE) processes [C. Amoruso *et al*, PRL 2006]
- $d_f^{\mathrm{SLE}} = 1.276(1)$, with $\theta = -0.287(4)$ [A.K. Hartmann *et al*, PRB 2002]

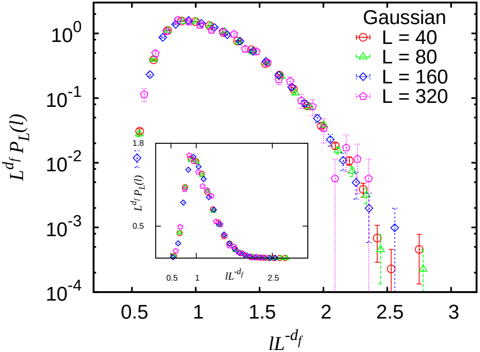
Fractal dimension of domain walls

- Scaling of DW roughness: $\langle r \rangle \sim L^{d_r}$, with $d_r = 1$



	d_r
Gaussian	1.008(3)
$\pm J^{\mathrm{min}}$	1.006(2)
$\pm J^{\mathrm{min2}}$	1.101(15)
$\pm J^{\mathrm{max}}$	0.993(2)

DW length



- Distribution $P_L(\ell)$ of DW lengths for gaussian disorder.
- One parameter scaling with $d_f = 1.274(1)$.
- Gaussian disorder: compares well with lognormal distribution.

Summary

- Groundstate study on 2D Ising spin glasses with short ranged interactions
- Minimum-weight path approach to the problem of finding DWs
- Fractal dimension of DWs for different types of disorder distributions
- Open: scaling of typical DWs for $\pm J$ disorder
- More details: OM and A.K. Hartmann, arXiv:0704.2004

Open Position

- Background in computational/statistical physics?
- Interested in a position in our Group:

CARL VON OSSIETZKY universität OLDENBURG

Computational Theroretical Physics
University Oldenburg

- 1 Phd position, apply now
- 1 Postdoc position, apply until 01.10.07
- For more information contact:

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