Reconstruction of the radial refractive index profile of optical waveguides from lateral diffraction patterns

Seminar der AG Computerorientierte Theoretische Physik, 28. Juni 2017

Gunnar Claußen[1]

[1]Fachbereich Ingenieurwissenschaften
Jade Hochschule am Standort Wilhelmshaven

OUTLINE

INTRODUCTION AND MOTIVATION (I)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Motivation

- **Fibres** in communication technology (waveguides)
- Distinct types: Gradient and step-index fibres
- Desired: Measurement and control of parameters (refractive index profile) during production, i.e. the drawing process

Current knowledge

- Methods exist for the determination of the outer diameter
- These methods work well for **intransparent** cylinders
- No algorithm is known for the refractive index profile $n(d)$ or $\{d_j, n_j\}$

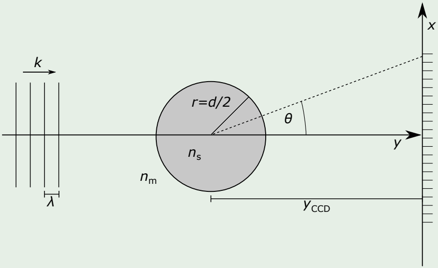
INTRODUCTION AND MOTIVATION (II)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

General approach

- Illumination of a cylinder with plane-wave incidence, $\zeta = 90^\circ$
- Refraction on layer boundaries and diffraction on edges
- Diffraction pattern is measured through an array of CCD cells
- Evaluation of the diffraction pattern for parameter determination

Experimental setup



Please note

- Setup is the same for stratified cylinders
- Distance y_{CCD} has to be known accurately
- All effects add up to scattering

INTRODUCTION AND MOTIVATION (III)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Theoretical approach: Inverse problem

- Direct problem: Parameters known, result unknown
- Inverse problem: Result known, parameters unknown

Illustration of an inverse problem in [Bohren & Huffman 1983]

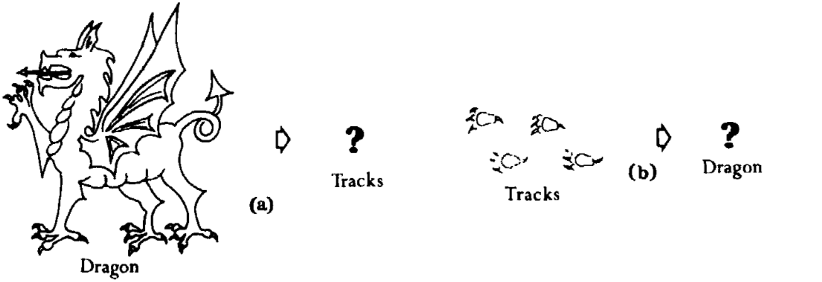


Figure 1.5 (*a*) The direct problem: Describe the tracks of a given dragon. (*b*) The inverse problem: Describe a dragon from its tracks.

MIE THEORY FOR CYLINDERS (I)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Light scattering: Basics of Mie theory

- Propagation of EM waves is expressed through Maxwell equations
- Especially: Vector wave equation $\nabla^2 \vec{E} + k^2 \vec{E} = 0$ and continuity at boundaries, $[\vec{E}_2 - \vec{E}_1] \times \hat{n} = 0$
- Mie theory: Transformation to specific coordinates corresponding to geometry allows **expansion of scattered fields into Bessel functions**

Scattering on cylinders

- Incident wave: $\vec{E}_i = \sum_{n=-\infty}^{\infty} E_n \vec{N}_n^{(1)}$
- Scattered wave: $\vec{E}_s = \sum_{n=-\infty}^{\infty} E_n \left[b_{nI} \vec{M}_n^{(3)} + i a_{nI} \vec{N}_n^{(3)} \right]$
- For normal incidence ($\zeta = 90°$), only z-components of $\|$-polarized components are relevant: $$E_{s\|}(z) = - \sum_{n=-\infty}^{\infty} E_n b_{n\|} N_n(z)$$

MIE THEORY FOR CYLINDERS (II)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Scattering coefficients

- General calculation for homogeneous ($r = 1$) or stratified ($r > 1$) cylinders:

$$b_n = \frac{m_r J_n(x_r)[J_n'(m_r x_r) + T_n^{r-1} N_n'(m_r x_r)] - J'n(x_r)[J_n(m_r x_r) + T_n^{r-1} N_n(m_r x_r)]}{m_r H_n(x_r)[J'n(m_r x_r) + T_n^{r-1} N_n'(m_r x_r)] - H_n'(x_r)[J_n(m_r x_r) + T_n^{r-1} N_n(m_r x_r)]} \quad (1)$$

$$T_n^s = \frac{m_s J_n(x_s)[J_n'(m_s x_s) + T_n^{s-1} N_n'(m_s x_s)] - J_n'(x_s)[J_n(m_s x_s) + T_n^{s-1} J_n(m_s x_s)]}{m_s J_n(x_s)[J_n'(m_s x_s) + T^{s-1} N_n'(m_s x_s)] - N_n'(x_s)[J_n(m_s x_s) + T_n^{s-1} J_n(m_s x_s)]} \quad (2)$$

- Recursive calculation: Layers $s = 1 \ldots r$, start with $T_0 = \{0\}$
- Depends only on the relative refractive index $m_s = \frac{n_s}{n_{s+1}}$ and the "size parameter" $x_s = k_{s+1} \cdot r_s$ of the s-th layer

Outline of the calculation of the diffraction pattern

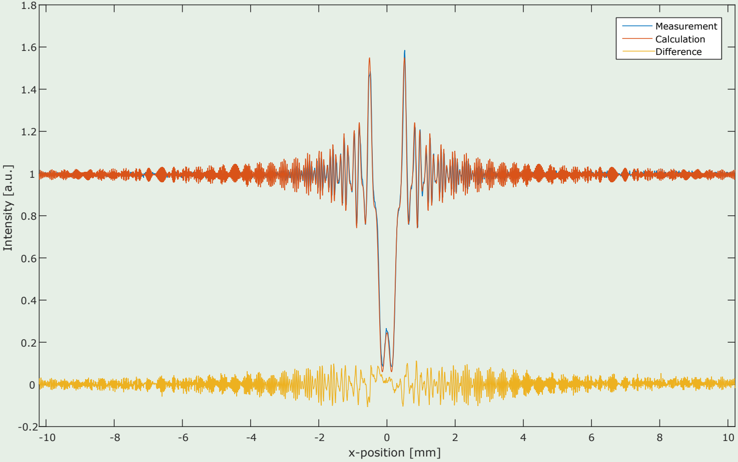
1. Calculate scattering coefficients b_n up to $n_{\max} = \lceil x + 4 \cdot \sqrt[3]{x} + 2 \rceil$
2. Transform CCD array to polar coordinates $(x, y) \mapsto (\rho, \varphi)$
3. Calculate vector-harmonic generating functions $N_n(z)$ for these coordinates
4. Calculate sum $E_{s\|}(z) = -\sum_{-n_{\max}}^{n_{\max}} E_n b_{n\|} N_n(z)$.

MIE THEORY FOR CYLINDERS (III)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

...and it works!

Agreement for an intransparent $d = 370\ \mu$m cylinder, $\lambda = 632.8$ nm



IRGN ALGORITHM (I)

Inverse problem (now formally...)

- Generally, a **operator** $\mathcal{F}$ on a **parameter set** r creates (maps to) a **far-field pattern** u_∞, i.e. a simple operator equation:

$$\boxed{\mathcal{F} : r \mapsto u_\infty} \quad \text{or} \quad \boxed{\mathcal{F}(r) = u_\infty} \tag{3}$$

- **Direct** problem: $\mathcal{F}$ and r are known, u_∞ is to be determined
- **Inverse** problem: $\mathcal{F}$ and u_∞ are known, r is sought

Solution of the inverse scattering problem

- Solution through the iteratively regularized Gauss-Newton (IRGN) algorithm:

$$\boxed{||\mathcal{F}'[r_n]h_n + \mathcal{F}(r_n) - u_\infty^\delta||^2 + \alpha_n||h_n + r_n^\delta - r_0||^2 \overset{!}{=} \min} \tag{4}$$

- In each iteration n: Calculate alteration h_n of the parameter set r_n
- Iteration terminates after N steps, if $||\mathcal{F}(r_N) - u_\infty^\delta||_2 \leq \tau\delta$

IRGN ALGORITHM (II)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

Regularisation

- Special treatment for noisy data, weighting of far-field pattern
- The step update h_n of parameters r_n is calculated according to

$$h_n = -\frac{F'[r_n^\delta]^*(F(r_n^\delta) - u_\infty^\delta) + \alpha_n(r_n^\delta - r_0)}{F'[r_n^\delta]^* F'[r_n^\delta] + \alpha_n I} \tag{5}$$

with regularisation parameter $\alpha_n = \alpha_0^n, \alpha_0 = 1/2$

Notes

- Derivative $\mathcal{F}'[r_n]$ to the parameters r_n has to be known
- For homogeneous cylinders: Analytcally...
- For stratified cylinders: Numerically...
- Analytical calculation does *not* pay off via a considerable time advantage!

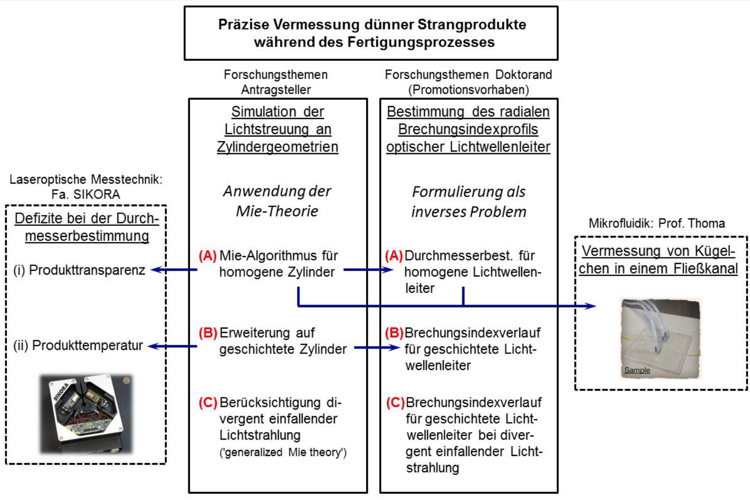
THE RESEARCH PROJECT (I)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

General properties

- Research project at Jade Hochschule
- Applied and carried out by Prof. Dr. Werner Blohm since 2014
- Aim: Improvement of the precision of diffraction-optics methods
- Doctoral research is embedded in the project through **Jade2Pro**
- Several related "milestones" have been proposed for both the applicant (Prof. Blohm) and the doctoral researcher (me)
- "Applicant" side ended in 2016, but research will be supported further

Structogram of the Project



THE RESEARCH PROJECT (III)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

Formal development of the doctorate

- Cooperation with Zentrum für Technomathematik (ZeTeM), Fachbereich 3, Bremen University
- Supervision by Prof. Dr. Armin Lechleiter, head of AG Inverse Probleme
- Guest in Bremen in WS 2015/16, seminar talk
- Beginning of WS 2016/17: Ph.D. student at Bremen
- In WS 2016/17: **Recognition of the research topic** by FB3
- Complete Title: "*Bestimmung des radialen Brechungsindexprofils optischer Lichtwellenleiter aus lateralen Streulichtverteilungen*"
- In WS 2016/17: Project **successfully evaluated** by Jade2Pro

Project-related "publicity" so far

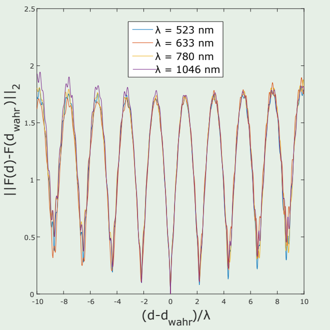
- Talks at 2nd, 3rd und 5th Jade2Pro-Kolloquium, Oldenburg
- Poster presentation at the DPG Spring Meeting 2017 of the Atomic, Molecular, Plasma Physics and Quantum Optics Section (SAMOP), Mainz

RESULTS: CYLINDER (I)

JADEHOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

The actual inverse scattering problem

- Operator $\mathcal{F}$: Mie theory for cylinder scattering
- Parameters r: Only diameter d, everything else is fixed $(n_s, n_m, y_{\text{CCD}}, \lambda)$
- Far-field u_∞: Pattern on CCD array at distance y_{CCD}
- Fréchet derivative $\mathcal{F}'$ for d is analytically known



Encountered "problem"

- Over d, the patterns resemble each other
- An IRGN algoritum iteratively minimizes the residual $||\mathcal{F}(d) - u_\infty||_2$
- The residual, when compared to a reference d_{real}, exhibits local minima...
- **Ill-posed problem** (i.e. non-bijective)
- **Danger of false convergence**

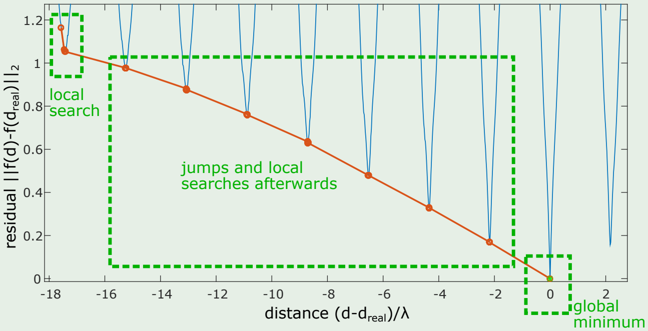
RESULTS: CYLINDER (II)

JADEHOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

"Solution": Modification of the IRGN algorithm

- But: Minima are spaced **periodically** by $\delta_{\min,s} \approx 2.18\lambda$
- This property may be utilized...
  $\Rightarrow$ Detect local convergence and then "jump" by $\pm 2.18\lambda$

Example for such a way of the algorithm



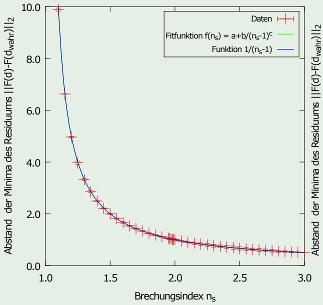
RESULTS: CYLINDER (III)

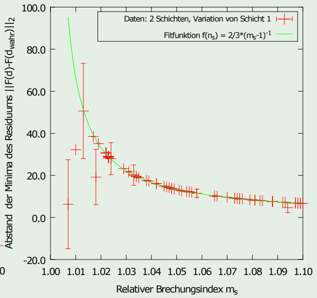
JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Where does the periodicity $\delta_{\min,s} \approx 2.18$ stem from?

- In the present case, $n_s = 1.458 \equiv m_s = \frac{n_s}{n_m} \ldots$
- Research: Compare diffraction patterns for a range of d with a reference d_{ref} for several values of m_s, measure minima spacing...

Dependence of minima spacing





Result

$$\delta_{\min,s} = \frac{1}{m_s - 1}$$

...thus:

$$\frac{1}{1.458-1} = 2.183\ldots$$

RESULTS: CYLINDER (IV)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

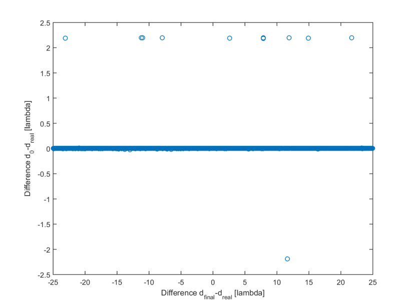
Ideas for further improvement

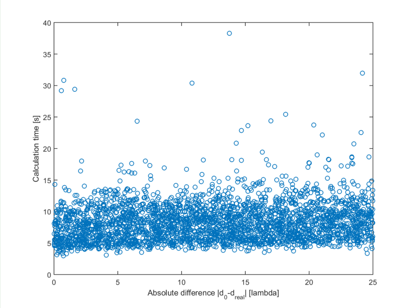
- Improvement of initial values: Sample for a few values of d around d_0, choose value of least residual
- **Approximation through intransparent cylinders**:
  - Subtraction of the 3rd sinusoid component from the diffraction pattern approximates the pattern for a intransparent cylinder of the same d
  - For intransparency, only a single global minimum exists...
  - GN iterations allow an approximation of d_0 to d_{real}
  - **Exact** determination of d_{real} is however not possible this way
- "**Plausibility check**": Are minima also present at d_N for further observing angles θ? Otherwise, it's certainly no reliable result...

RESULTS: CYLINDER (V)

Precision of the algorithm: Numerical evaluation

- $n = 1.4$, $N = 2910$ random $d_{\text{real}} = 110 \ldots 150\,\mu$m and $d_0 \in d_{\text{wahr}} \pm 25\lambda$



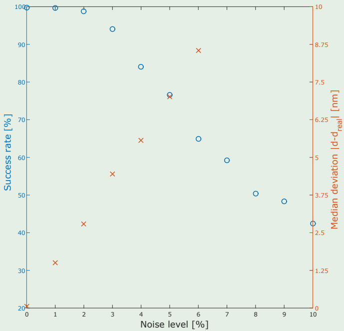


- The actual diameter d_{real} is met precisely (i.e. $|d_N - d_{\text{real}}| < 0.1\,\mu$m, definition of a "success") for 99.62% of the cases
- The calculation time exhibits no visible dependence on the distance between initial and actual values of d

RESULTS: CYLINDER (VI)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Precision in presence of noise



- Noise: Multiplicative, normally distributed
- "Noise level": Scaling factor for the width (variance) of the distribution
- Deviation of the result calculated only for "successful" cases
- Note: "Failures" still finish within local minima

Summary

- Method for diameter determination is very precise
- Precision drops linearly with noise level
- Improvements lead to constant calculation times

STRATIFIED CYLINDER (I)

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Basics

- **Aim**: Extension of the IRGN algorithm for stratified cylinders
- Calculation of diffraction patterns: Only different scattering coefficients
- Experience from homogeneous cylinders: Periodic local minima

Expected difficulties

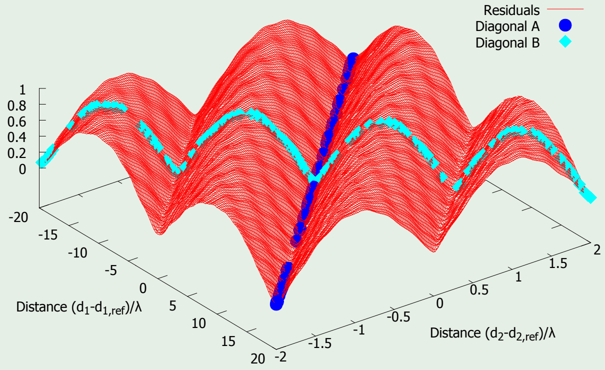
- Parameters to be found: Layer diameters d_j
- More complex operation, more parameters must be optimised
- Especially: How can the "jumps" of the modified IRGN algorithm be implemented?

STRATIFIED CYLINDER (II)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

Example: $J = 2$ layers, $d_{\text{ref}} = [50, 120]\ \mu$m, $n = [1.55, 1.5]$

Periodic behaviour and a global minimum at $d \equiv d_{\text{ref}}$ exist



STRATIFIED CYLINDER (III)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

Further findings

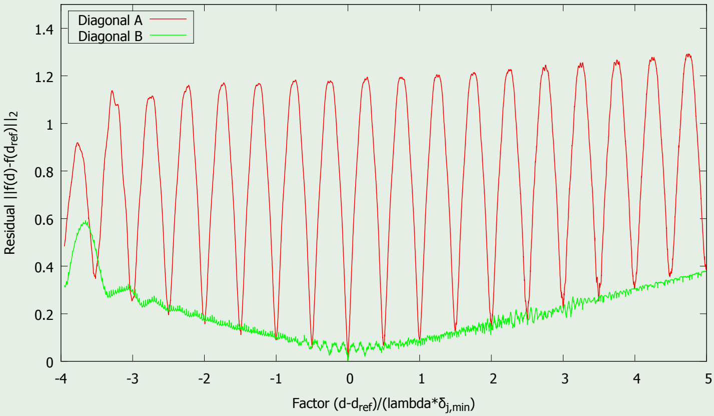
- Periodicity of minima for layer $j < J$: $\boxed{\delta_{\min,s} = \frac{2}{3}\frac{1}{m_s - 1}}$
- Remark: Refractive indices differ only marginally between layers, $|n_j - n_{j+1}| = 0.001...0.02 \Rightarrow n_{j+1} = 1.5 : m_j \approx 1.000\overline{6}...1.01\overline{3}$
- Thus: For inner layers "large" periodicity can be expected, $\delta_{\min,s} \approx 50....1000\lambda$
- For the outermost layer $j = J$ the jump from n_J to n_m is much larger $\Rightarrow$ cf. homogeneous cylinder

STRATIFIED CYLINDER (IV)

JADEHOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Even more further findings

The residuals along the diagonals in parameter space (d_1, d_2):



STRATIFIED CYLINDER (V)

JADE HOCHSCHULE Wilhelmshaven Oldenburg Elsfleth

Ideas for layer diameter determination

- IRGN steps lead into a "trench"
- The "trench" is linear and described through the periodicities $\delta_{j,\min}$
- Jumps should lead from one "trench" to another (diagonal A)
- Motion within one "trench": local Minima (diagonal B)?
- **Summary:** A combination of IRGN, jumping and sampling methods seems promising for determing both diameters d_1 and d_2
- ...and how can that be applied to $J > 2$?

FURTHER OUTLOOK

Aims to be reached

- **Milestone**: Algorithm for stratified cylinders is to be completed
- Experimental verification for homogeneous cylinder (experiment exists)
- Optionally: Generalized Lorenz-Mie theory for spherical waves

Temporal constraints

- Current position is running until April 2018
- Probably extended by one year from remaining funds
- Current difficulty: A. Lechleiter out of office until further notice

REFERENCES

References

- **Bohren, C.F.; Huffman, D.R.:** "Absorption and Scattering of Light by Small Particles", John Wiley & Sons, New York, 1983
- **Gerlinski, V.:** "Bestimmung geometrischer Parameter von Glasfasern aus Streulichtmustern", Master thesis, Universität Bremen, 2016
- **Quinten, M.**: "Optical Properties of Nanoparticle Systems. Mie and Beyond", Viley-VCH, Weinheim, 2011
- **Hohage, T.; Schormann, C.:** "A Newton-type method for a transmission problem in inverse scattering", Inverse Problems 14, 1207–1227, 1998
- **Schäfer, J.-P.:** "Implementierung und Anwendung analytischer und numerischer Verfahren zur Losung der Maxwellgleichungen für die Untersuchung der Lichtausbreitung in biologischem Gewebe", Ph.D. thesis, Ulm, 2011

JADE HOCHSCHULE
Wilhelmshaven Oldenburg Elsfleth

Thanks for your attention!