JACOBS UNIVERSITY

How few elements can systematically shape large-scale patterns

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Jacobs University Bremen

Computational
Systems Biology

Introduction

Key question

How can the properties of individual elements shape the collective behaviors ("patterns") in a system?

New layer of predictability in self-organized patterns

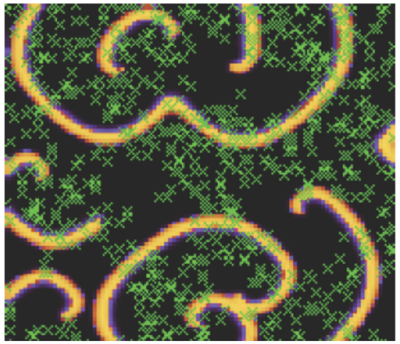
Excitable dynamics

- fundamental process: propagation of excitations through a system
- simple model of epidemic spread of infectuous diseases
- relevance to diverse biological processes
- relevance to a broad range of socio-economic processes
    - opinion formation
    - information spreading
    - any sort of cascade or wave phenomena

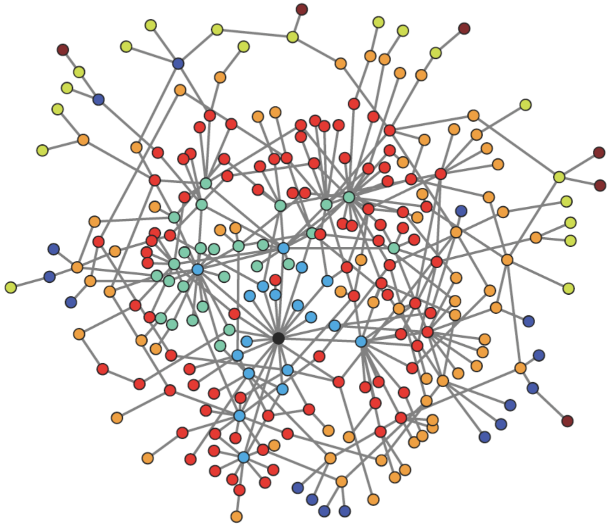
Introduction

Three examples

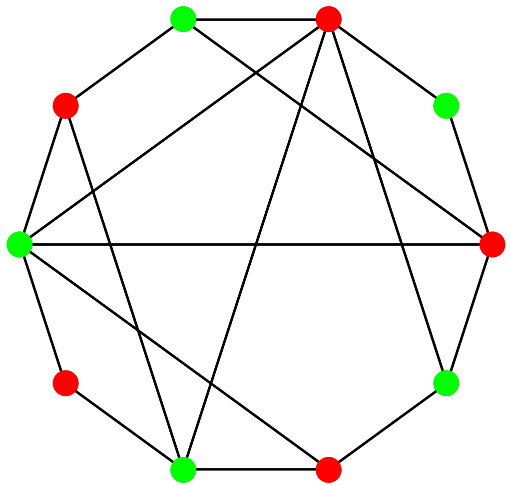
(1) small case study on biological pattern formation



(2) excitable dynamics on graphs

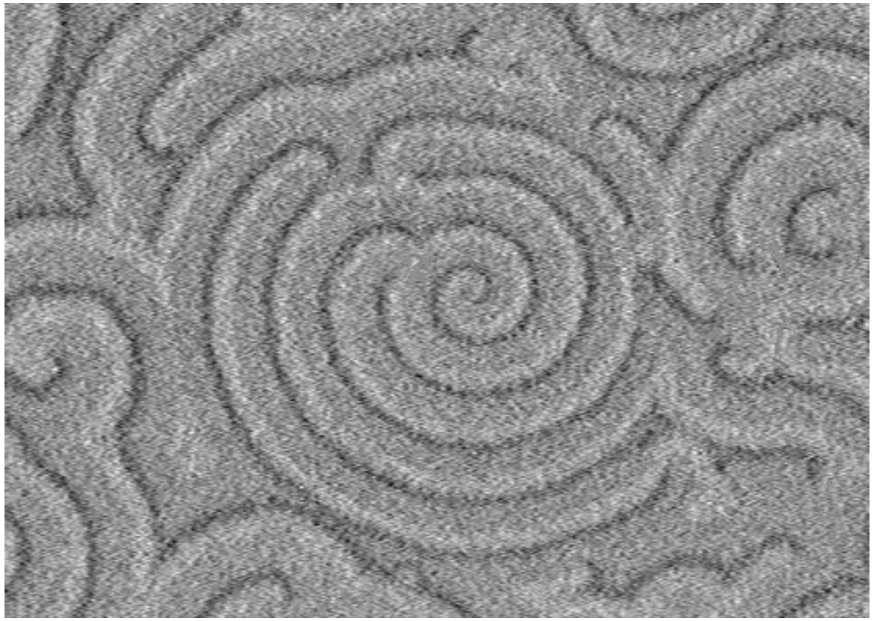


(3) an example of collective problem-solving

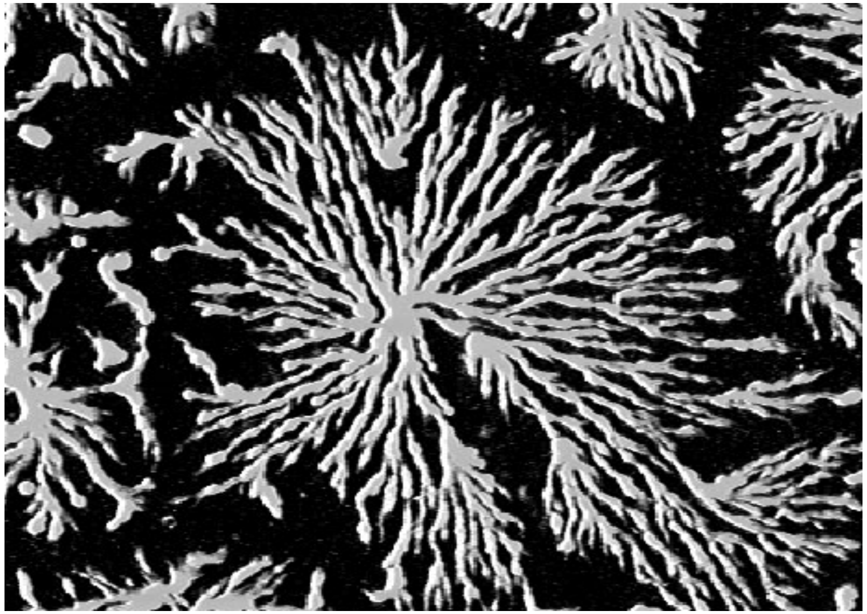


Spatiotemporal patterns

The model system



Dictyostelium discoideum: early-stage pattern



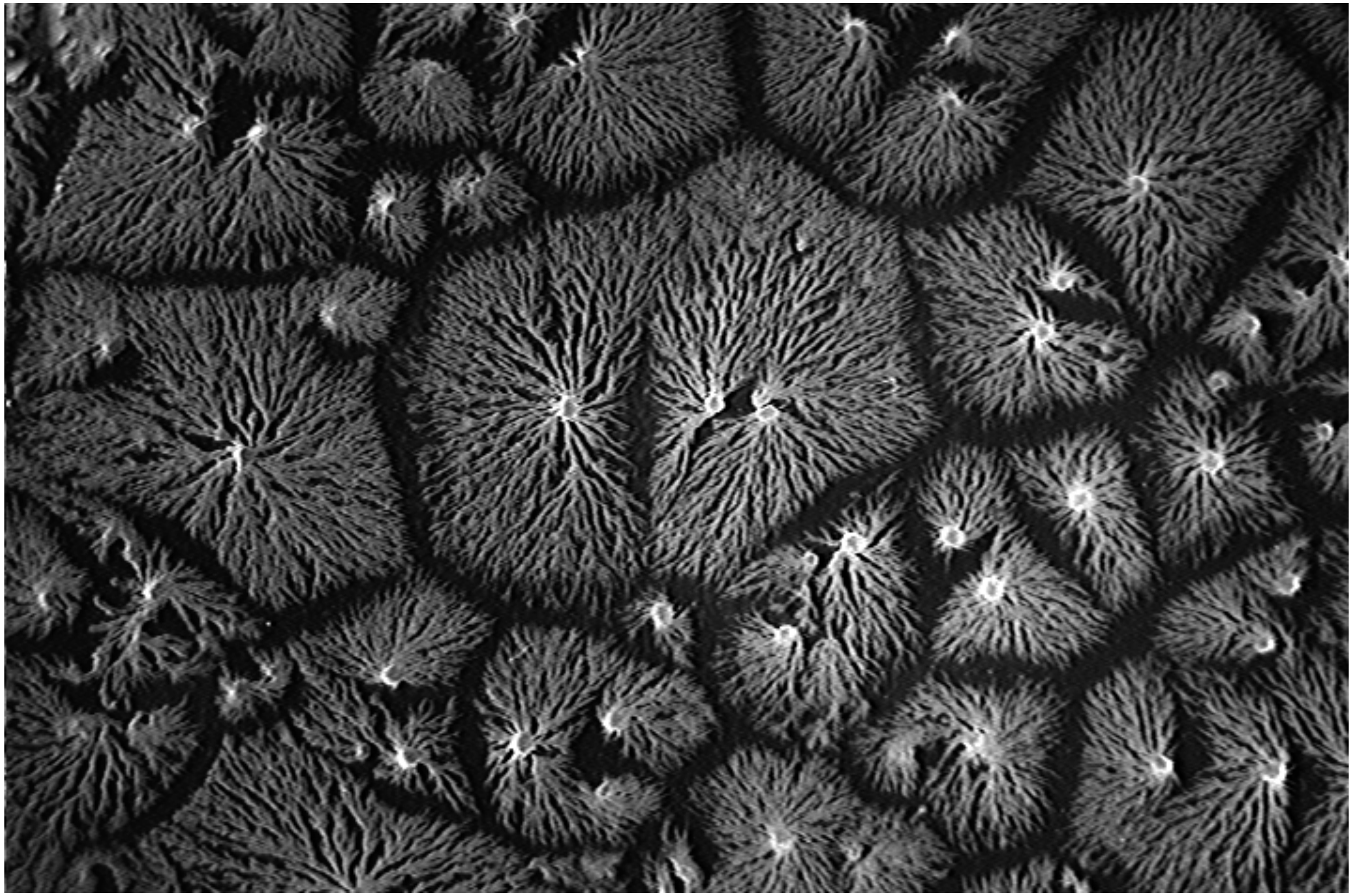
Dictyostelium discoideum: later-stage pattern

- remarkable collective problem-solving capacity
- optimized for a particular size of the collective states

Spatiotemporal patterns

The model system

'problem solving'

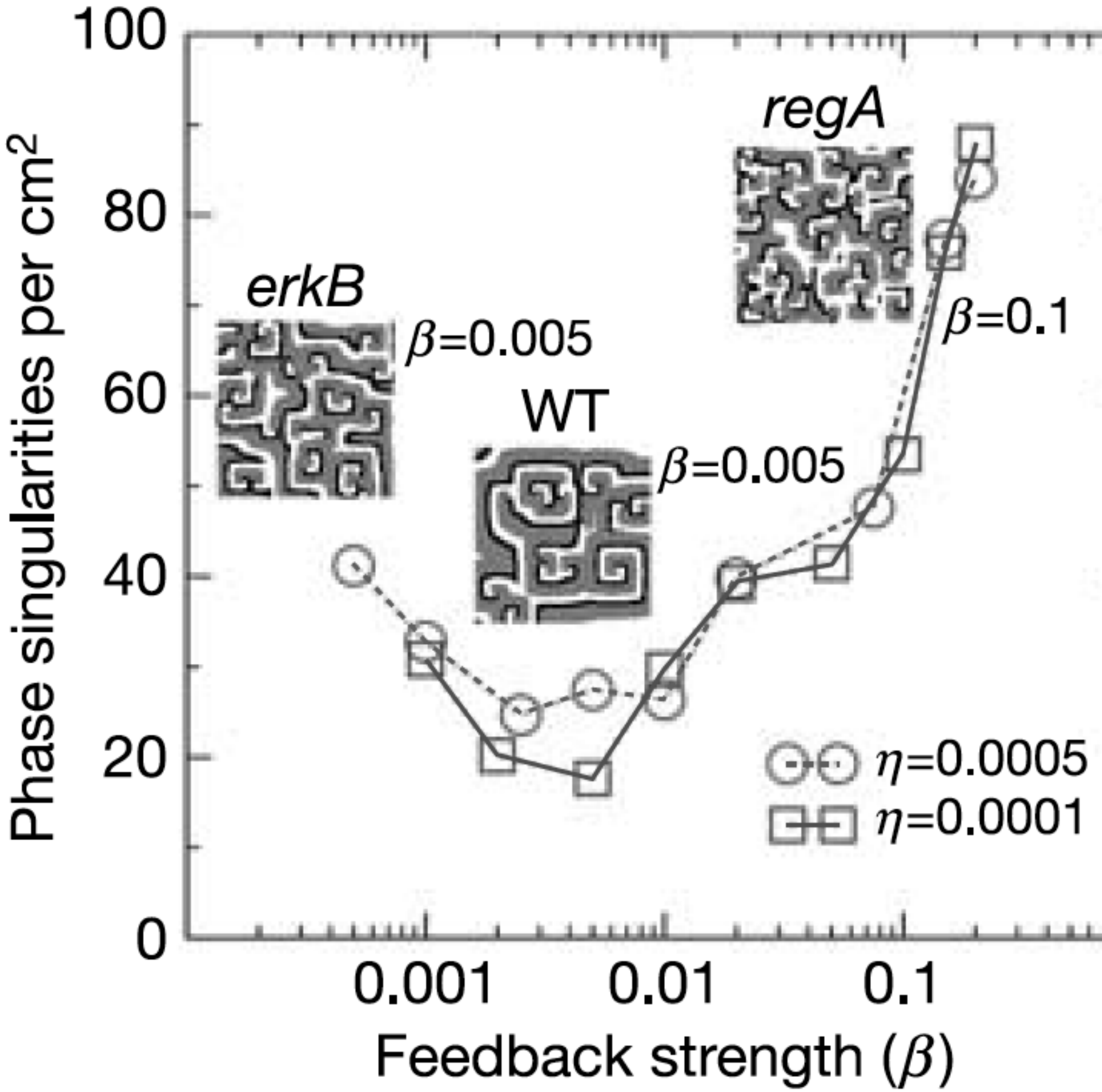


Dictyostelium discoideum

Spatiotemporal patterns

The model system

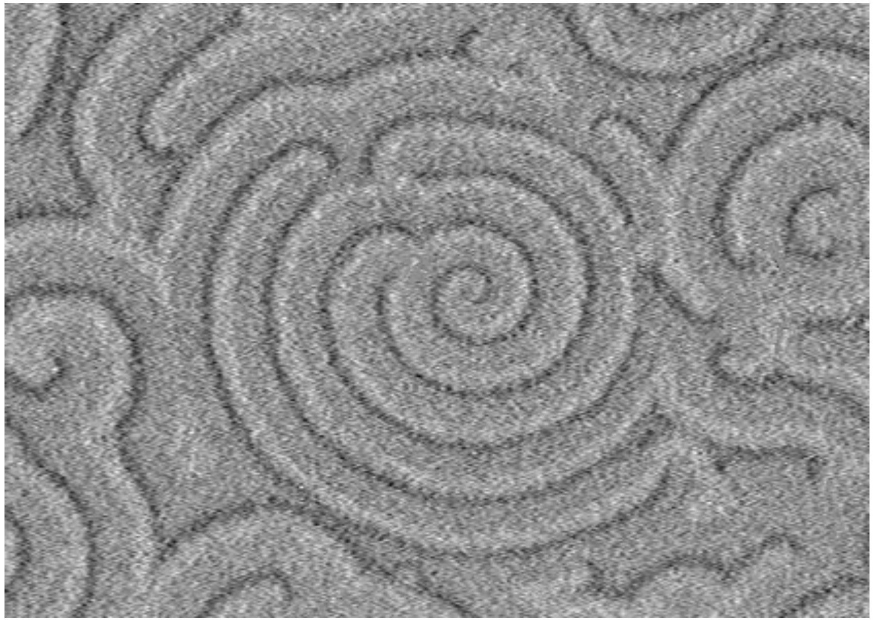
'size optimization'



Taken from: Sawai et al. (2005) Nature 433, 323

Spatiotemporal patterns

The model system



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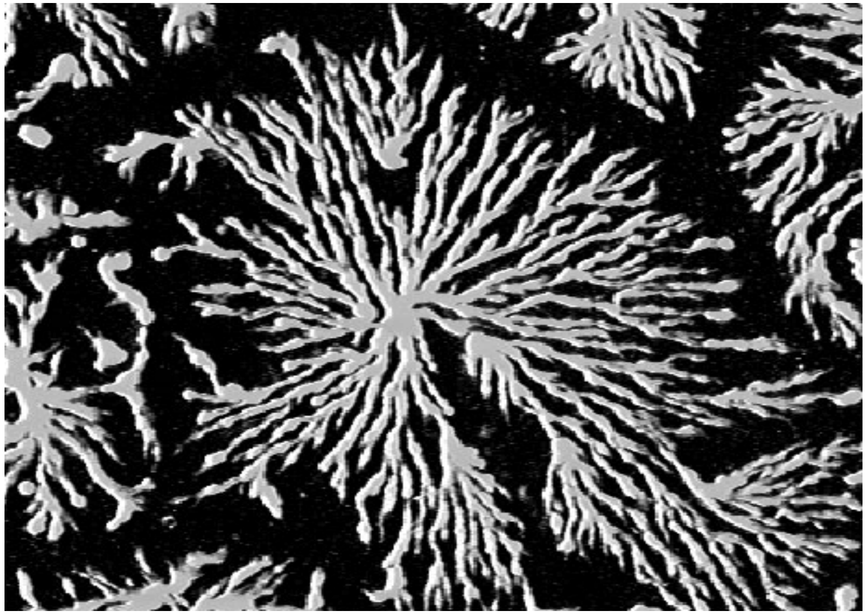


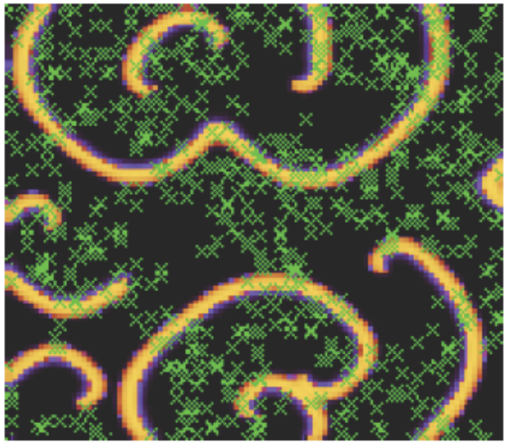
Dictyostelium discoideum: later-stage pattern

- remarkable collective problem-solving capacity
- optimized for a particular size of the collective states

- How do patterns start?
- What is the role of cellular variability?
- How does the distribution of cell properties translate into patterns?

Spatiotemporal patterns

Role of biological variability



simulation of spiral wave patterns for a distribution of pacemaker cells

refractory/quiescent cell: **s = 0**
firing cell: **s = 1**

- (dynamic) threshold for **c** decides over firing
- spontaneous firing of 'pacemaker cells'

cAMP concentration — **'state matrix'**

$$\dot{c}_{ij} = -\Gamma c_{ij} + r_F s_{ij}(t) + D L_{ij}(c)$$

$$\dot{E}_{ij} = \eta + \beta c_{ij}$$

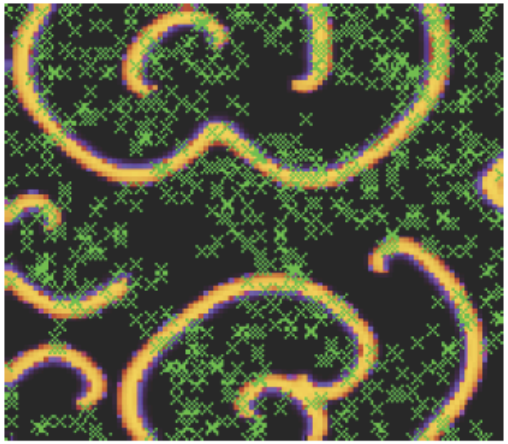
excitability — **feedback strength**

model from:
Levine et al. (1996) PNAS 93, 6382

Geberth and Hütt (2008) Phys.Rev. E 78, 031917

Spatiotemporal patterns

Role of biological variability



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Spatiotemporal patterns

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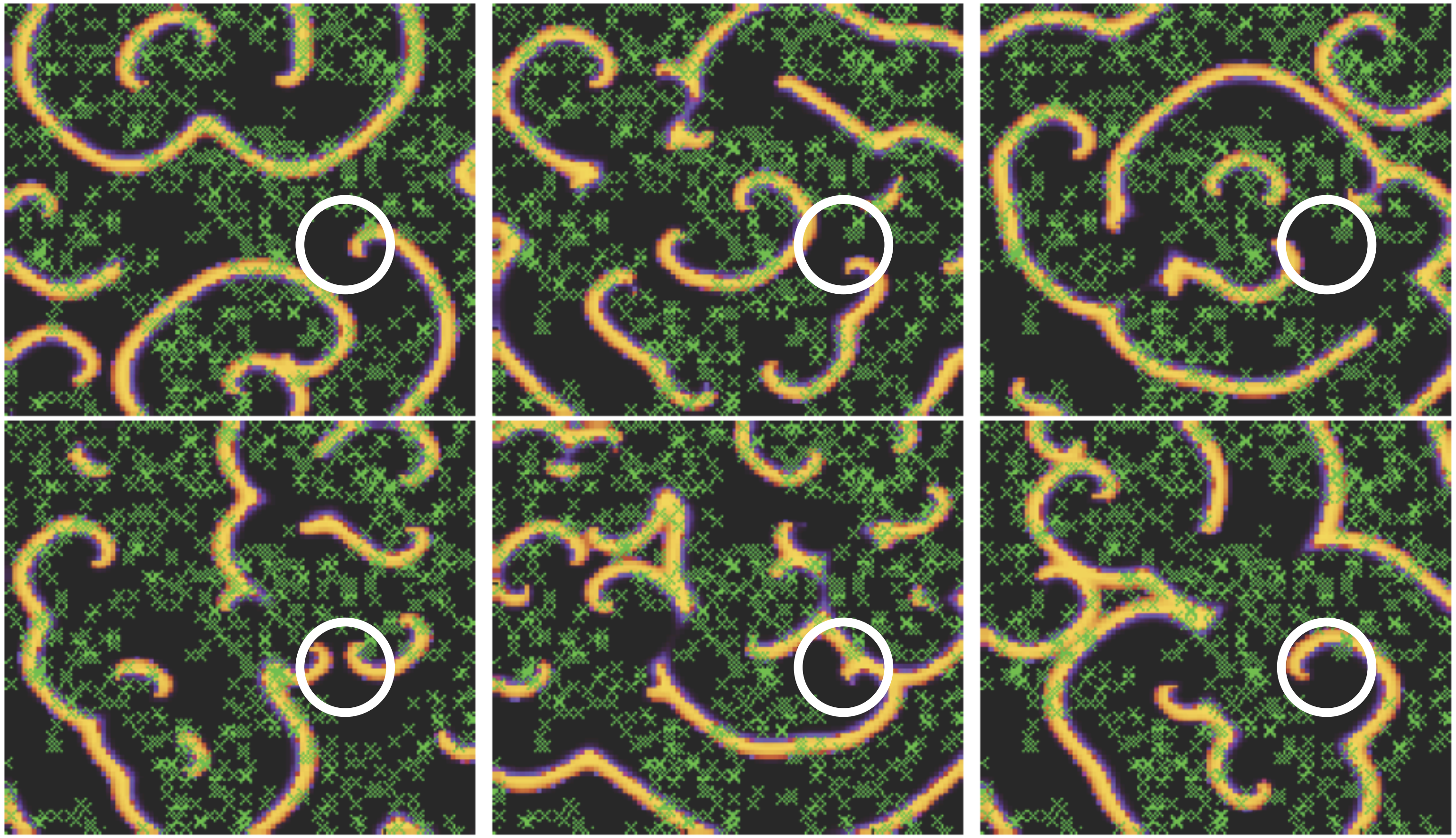


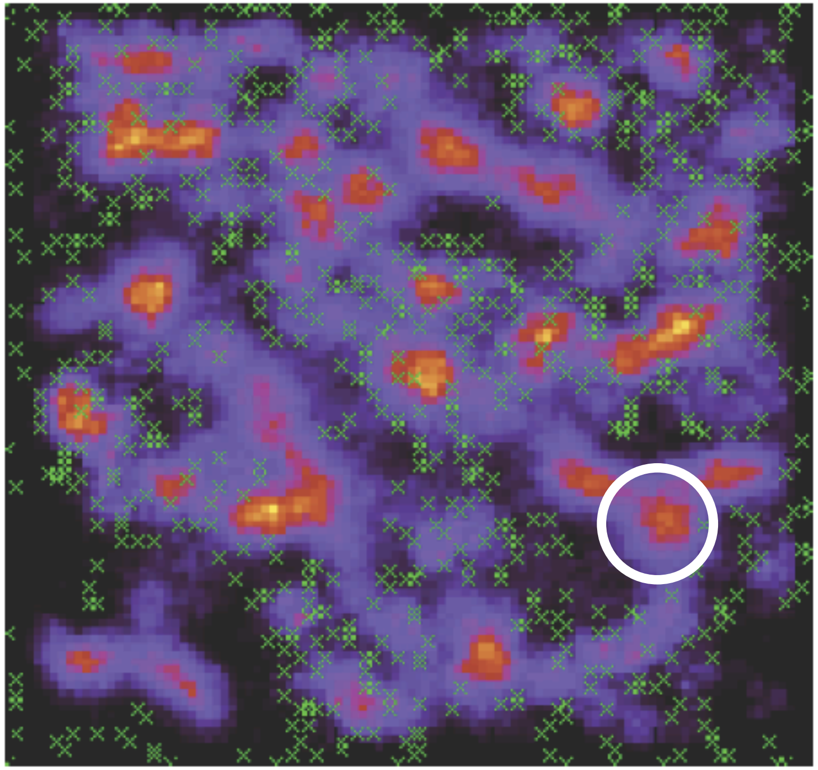
different simulation runs with same cell properties

model from
Levine et al. (1996)
PNAS 93, 6382

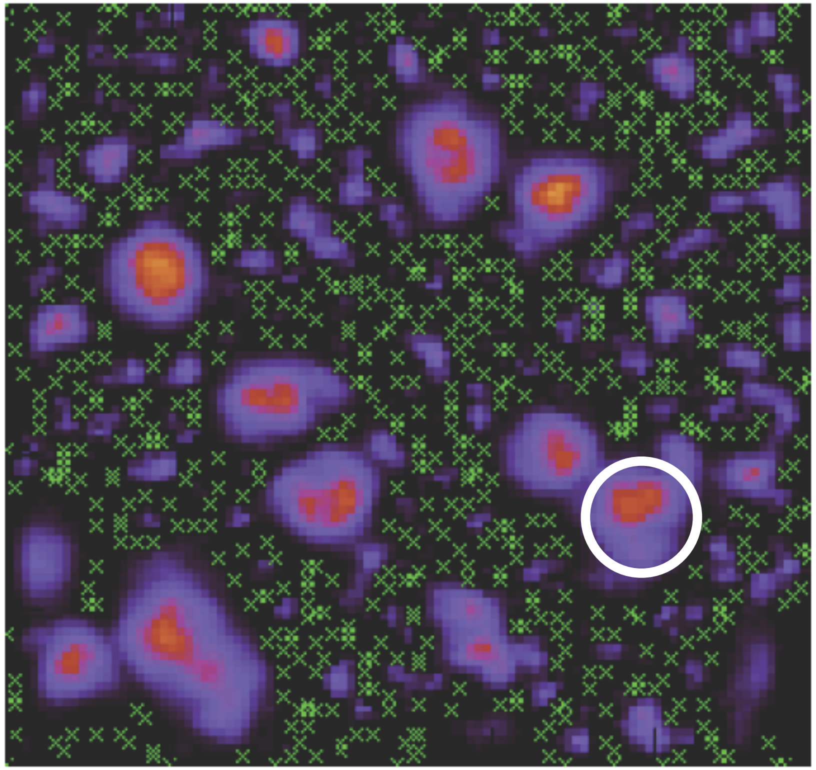
Geberth and Hütt (2008) Phys.Rev. E 78, 031917

Spatiotemporal patterns

Role of biological variability



distribution of spiral waves across 1000 simulation runs



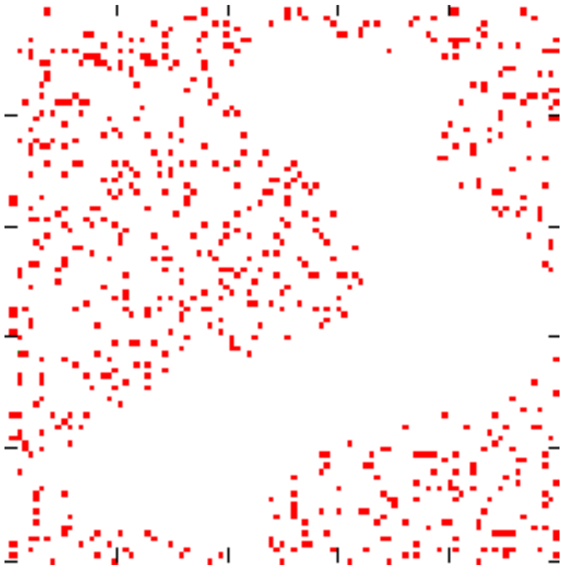
geometrical prediction

Geberth and Hütt (2008) Phys.Rev. E 78, 031917

Spatiotemporal patterns

Role of biological variability

prescribed pacemaker distribution



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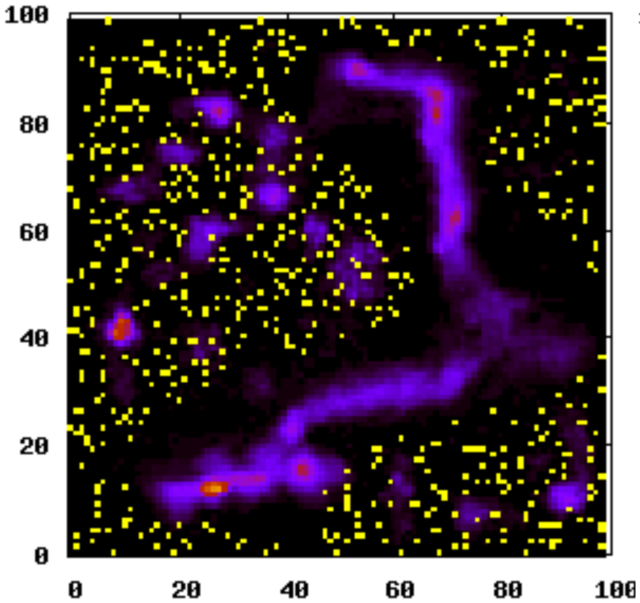
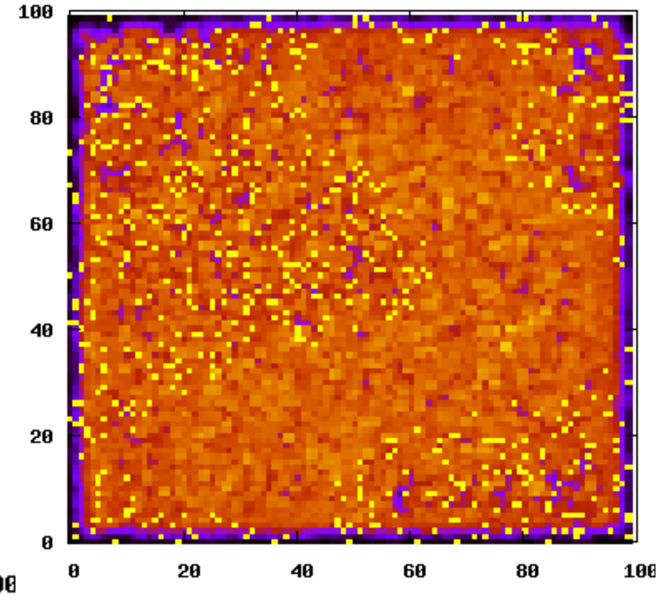


model from:
Lauzeral, Halloy, Goldbeter (1996) PNAS 94, 9153

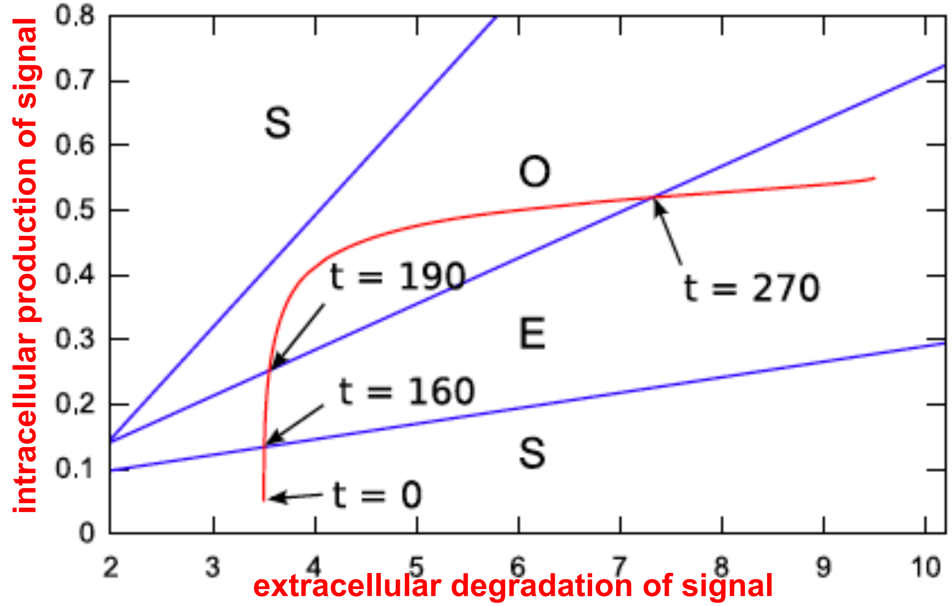


- anticorrelation of pacemaker cells and spiral waves
- some features can be understood in a simple geometric model
- alternative model fails to reproduce this anticorrelation

Geberth and Hütt (2008) Phys.Rev. E 78, 031917

Spatiotemporal patterns

A detailed look at the refined model



model from:
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PNAS 94, 9153

fraction of active receptors

$$\frac{d\rho_T}{dt} = -f_1(\gamma)\rho_T + f_2(\gamma)(1-\rho_T),$$

intracellular cAMP

$$\frac{d\beta}{dt} = q\sigma\Phi(\rho_T,\gamma,\alpha) - (k_i + k_t)\beta,$$

external cAMP

$$\frac{\partial\gamma}{\partial t} = (k_t\beta/h) - k_e\gamma + D_\gamma\nabla^2\gamma,$$

$$f_1(\gamma) = \frac{k_1 + k_2\gamma}{1+\gamma}, f_2(\gamma) = \frac{k_1L_1 + k_2L_2c\gamma}{1+c\gamma}$$

$$\Phi(\rho_T,\gamma,\alpha) = \frac{\alpha(\lambda\theta + \varepsilon Y^2)}{1+\alpha+\varepsilon Y^2(1+\alpha)}, Y = \frac{\rho_T\gamma}{1+\gamma}.$$

Spatiotemporal patterns

A detailed look at the refined model

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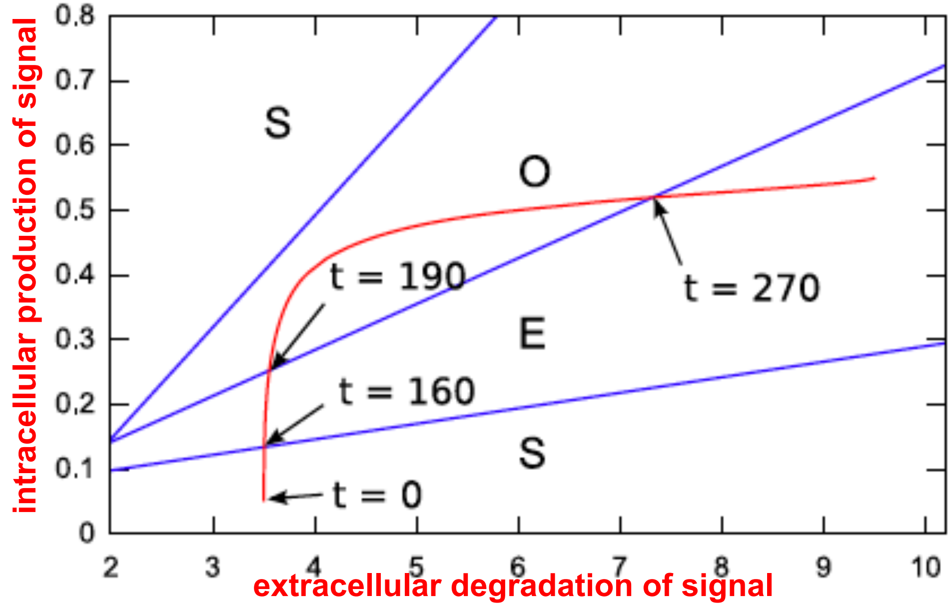
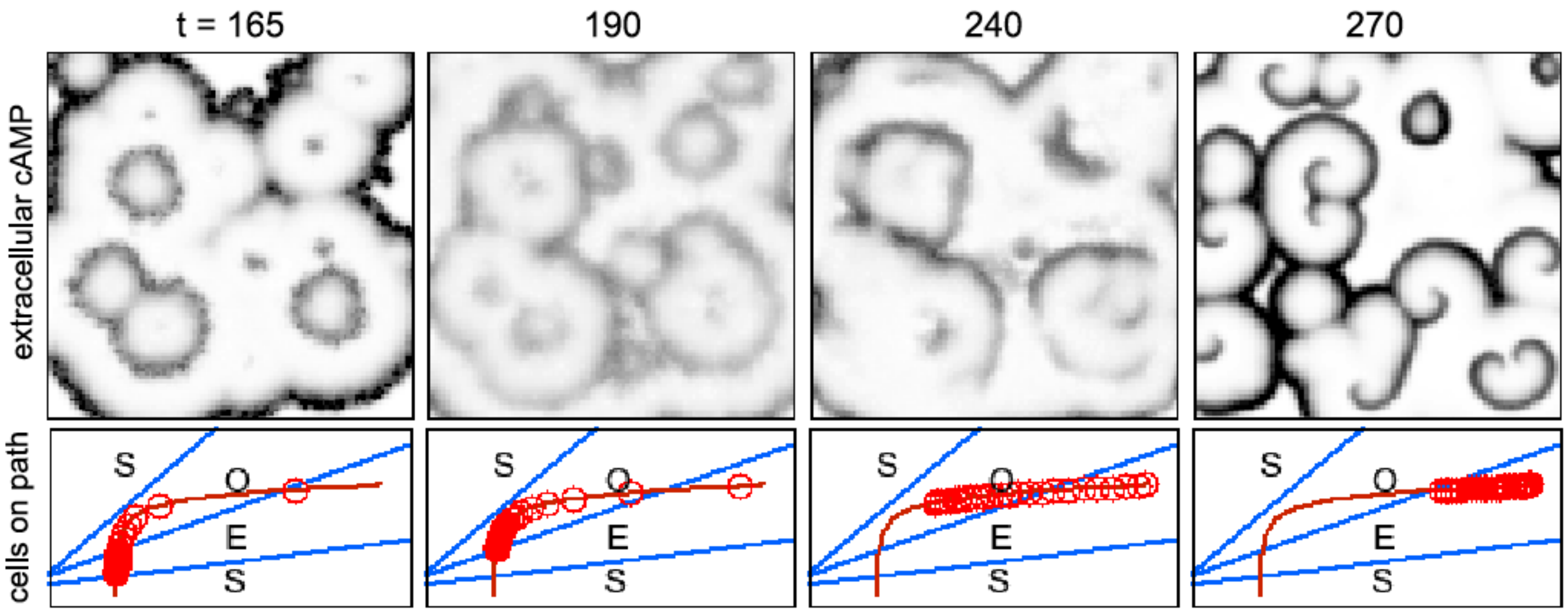
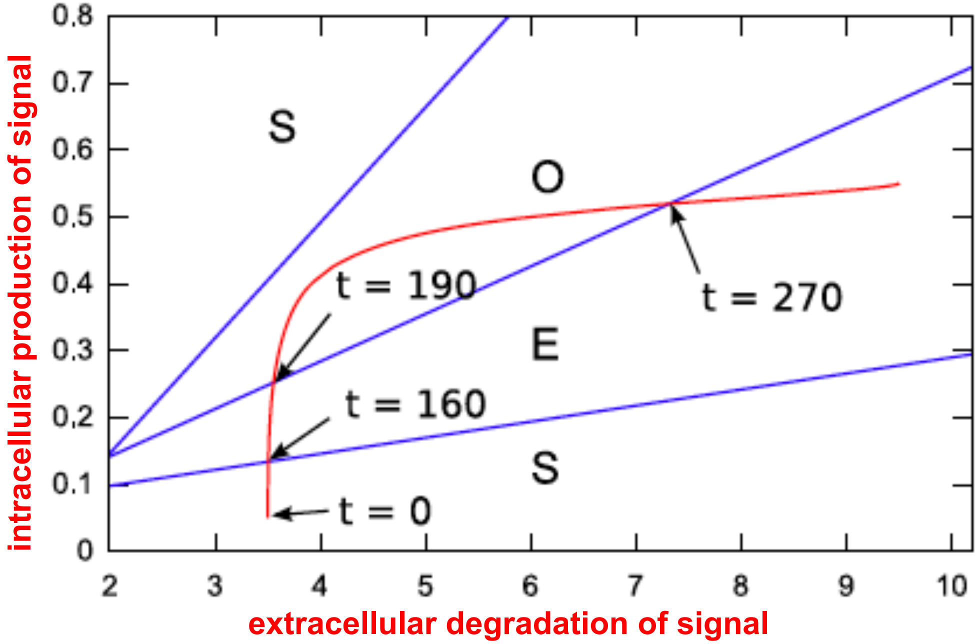




Spatiotemporal patterns

A detailed look at the refined model



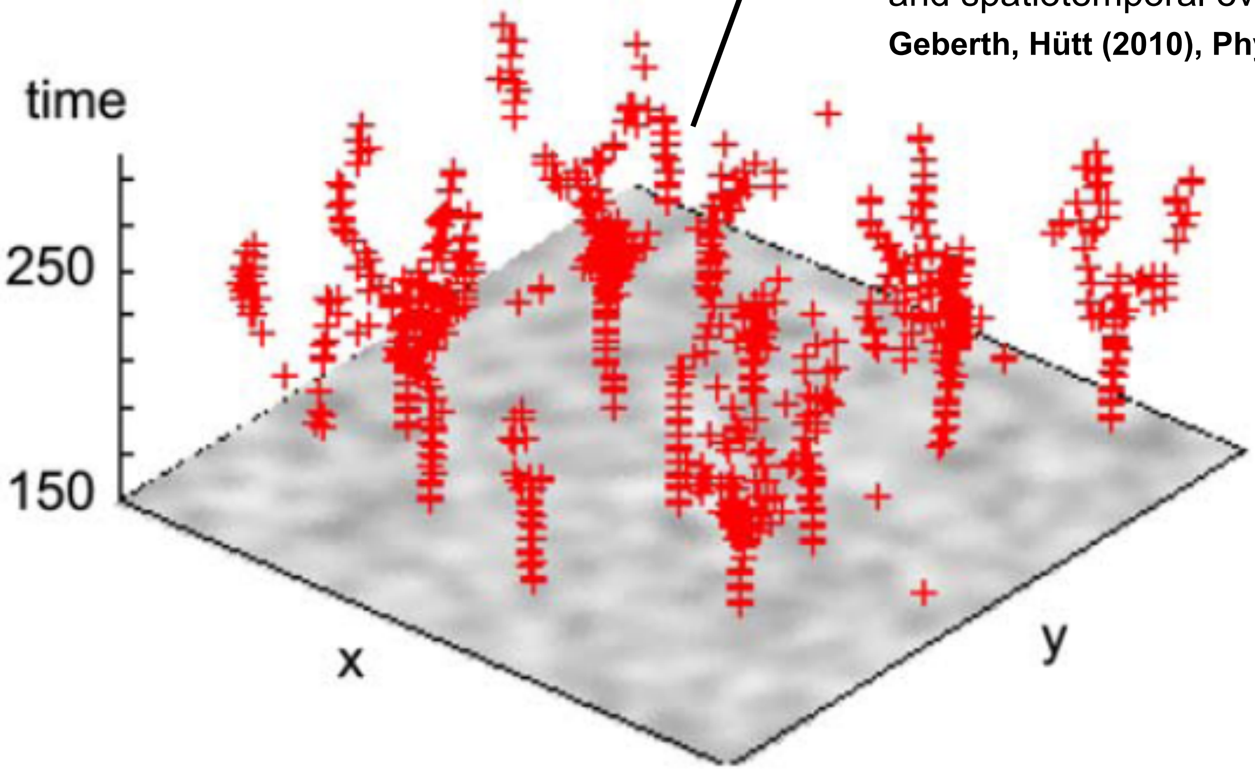
- **previous definition of a pacemaker cell:** all elements initially in the oscillatory regime
- **new, refined definition:** all elements in the oscillatory regime, when the last elements entered the excitable regime

Geberth and Hütt (2009) PLoS Computational Biology 5, e1000422

Spatiotemporal patterns

A detailed look at the refined model

Software package for pattern simulation and spatiotemporal event reconstruction
Geberth, Hütt (2010), Physica A 389, 249.

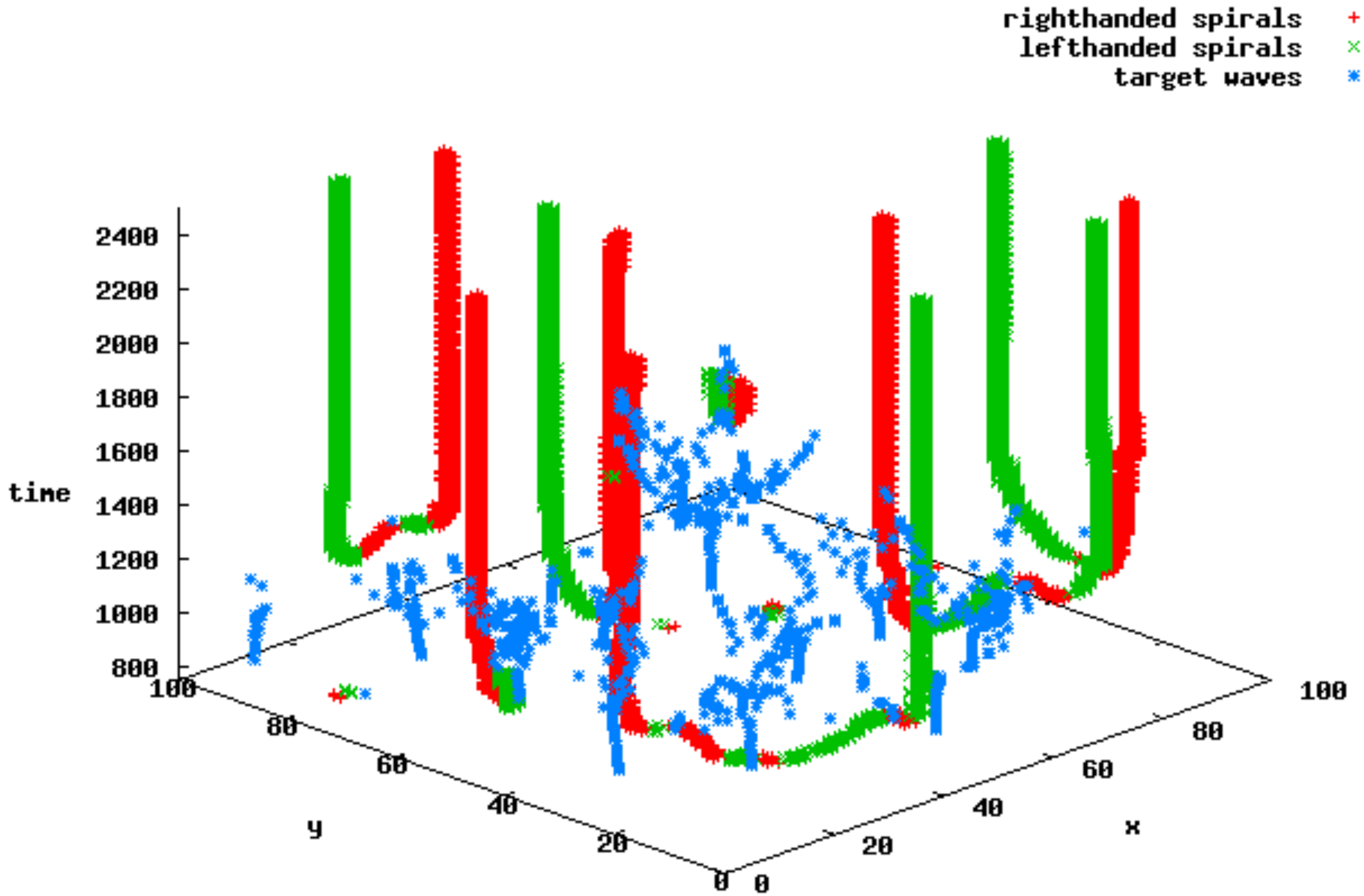


- **detected target wave events correlate with the refined 'pacemaker' cells**

Geberth and Hütt (2009) PLoS Computational Biology 5, e1000422

Spatiotemporal patterns

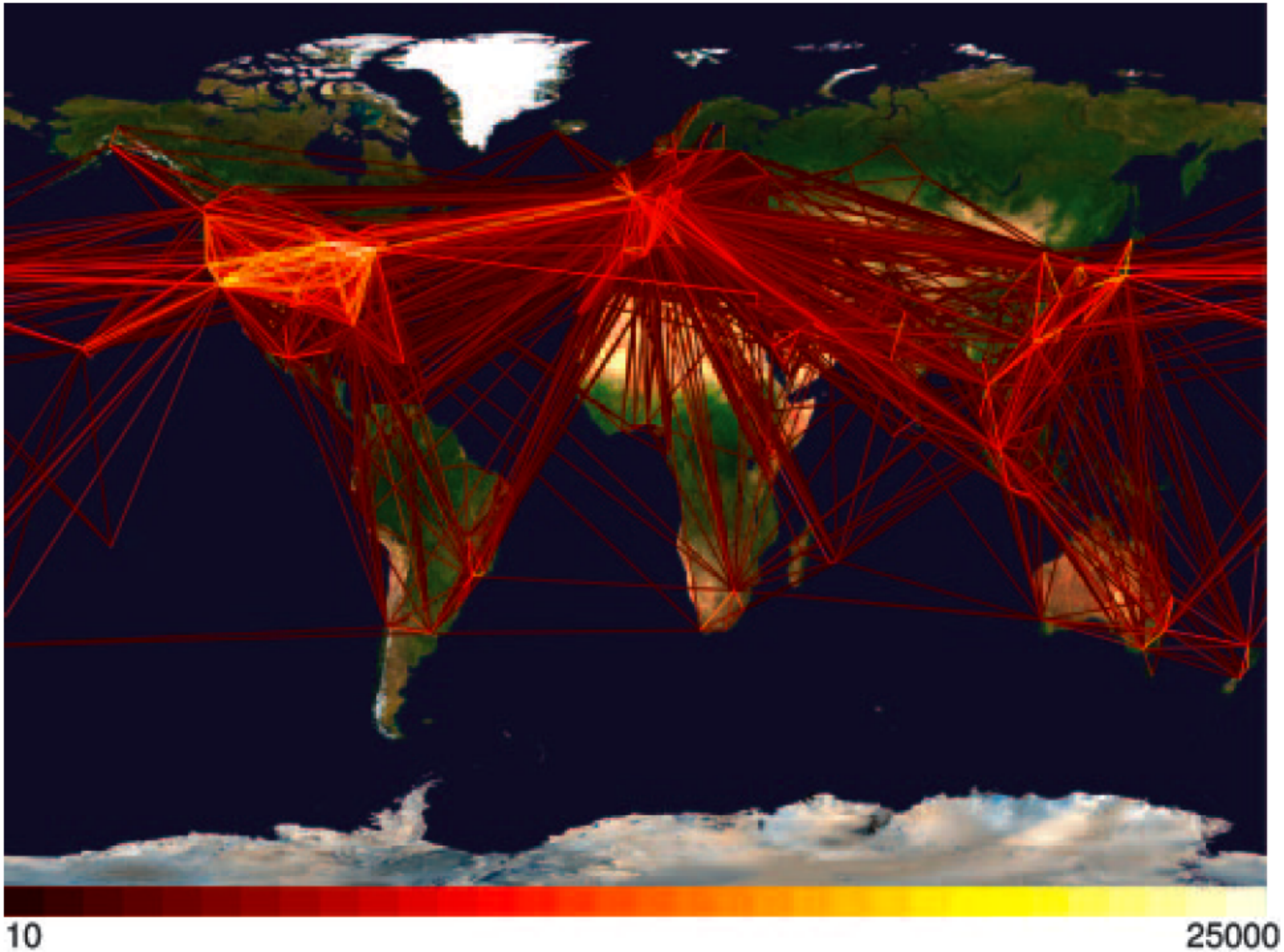
A detailed look at the refined model



Geberth and Hütt (2009) PLoS Computational Biology 5, e1000422

Excitable dynamics on graphs

Motivation



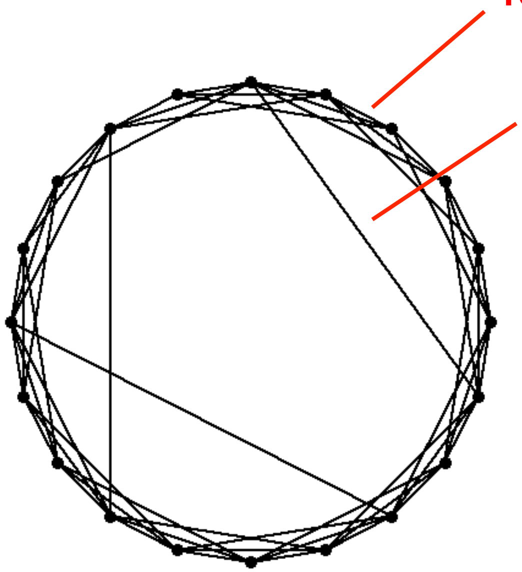
Taken from: Hufnagel et al. (2004) PNAS 101, 15124

Excitable dynamics on graphs

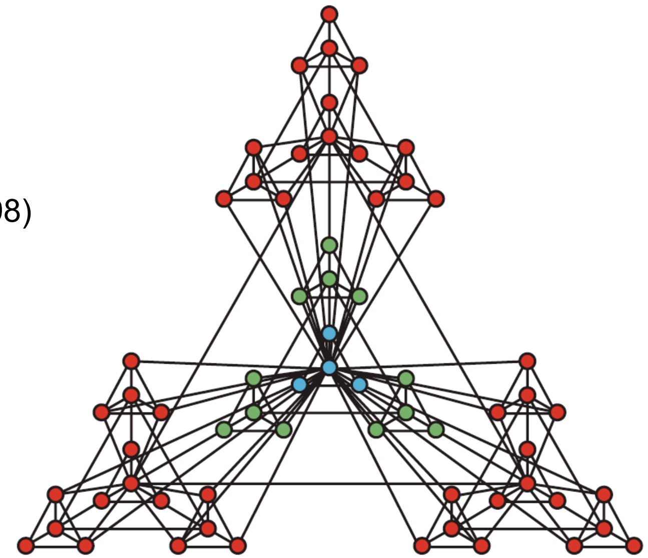
Motivation

- (at least) two distinct fields of research involved:
  - neural information processing
  - "pattern formation" aspects: heart cells, calcium dynamics, . . .
- abstract models help understand properties of such systems
- here we qualitatively link the "pattern" level with the "neural" level by studying pattern formation of excitable dynamics on graphs

Some previous findings



Watts and Strogatz (1998)
Nature 393, 410



hierarchical graph concept taken from:
Ravasz et al. (2002) Science 297, 1551

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Some previous findings

- percentage of shortcuts in small-world networks triggers a transition from activity failure to persistant activation for excitable integrate-and-fire neurons

  Roxin, Riecke, Solla (2004) Phys.Rev.Lett. 92 198101

- importance of hierarchical structures
    - in neural information processing

      Zhou et al. (2006) Phys.Rev.Lett. 97, 238103
      Kaiser and Hilgetag (2007) Neurocomputing 70, 1829

    - and synchronization

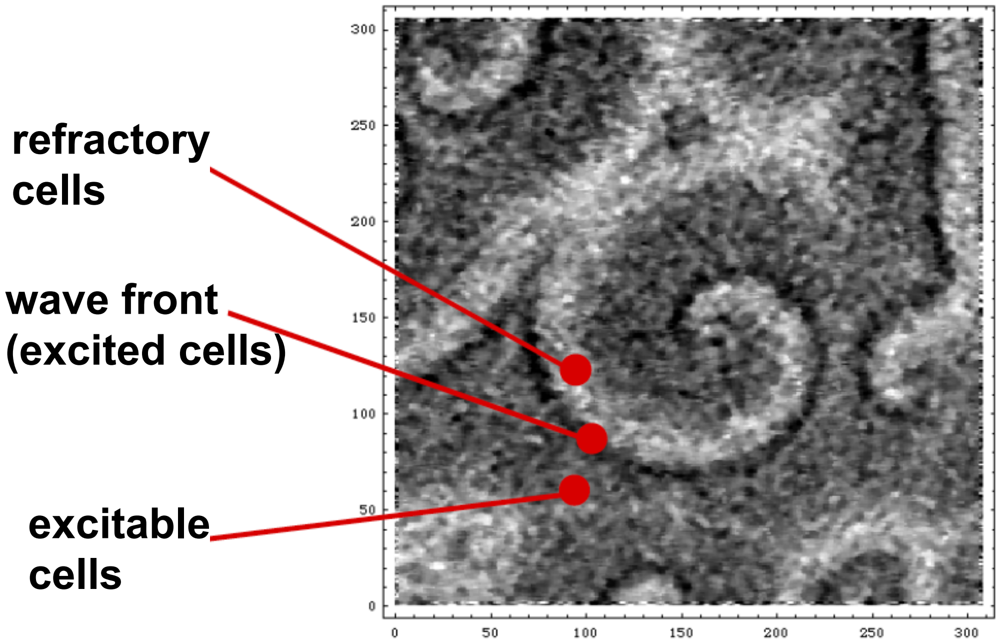
      Arenas et al. (2006) Phys.Rev.Lett. 96, 114102

- functional similarity of noise and shortcuts

  Graham and Matthai (2003) Phys.Rev.E 68, 036109
  Marr and Hütt (2006) Phys.Lett.A 349, 302

Excitable dynamics on graphs

A minimal model



Scheme

excitable → excited → refractory → refractory → excitable → . . .

excitation in the neighborhood (excitable → excited)

refractory time (refractory → refractory)

Excitable dynamics on graphs

A minimal model

$$A \longrightarrow R$$

$$E \xrightarrow{\exists A \, in \, NB} A$$

$$R \xrightarrow{p} E$$

$$E \xrightarrow{f} A$$

A: active
R: refractory
E: excitable

p: recovery rate
f: rate of spontaneous excitations

- classical model of sustained excitable dynamics
- "forest-fire" model

  Drossel and Schwabl (1992) Phys.Rev.Lett. 69, 1629
- has been studied previously on graphs

  Graham and Matthai (2003) Phys.Rev.E 68, 036109
  Carvunis et al. (2006) Physica A 367, 595
- similar to other three-state models (SIR etc.)

Excitable dynamics on graphs

A minimal model

$$A \longrightarrow R$$

$$E \xrightarrow{\exists A \, in \, NB} A$$

$$R \xrightarrow{p} E$$

$$E \xrightarrow{f} A$$

A: active
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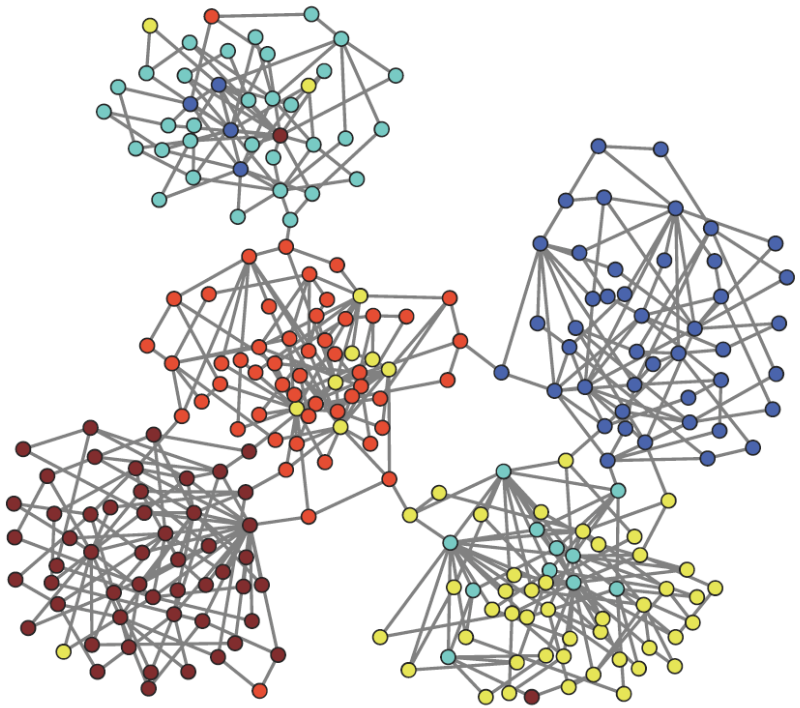
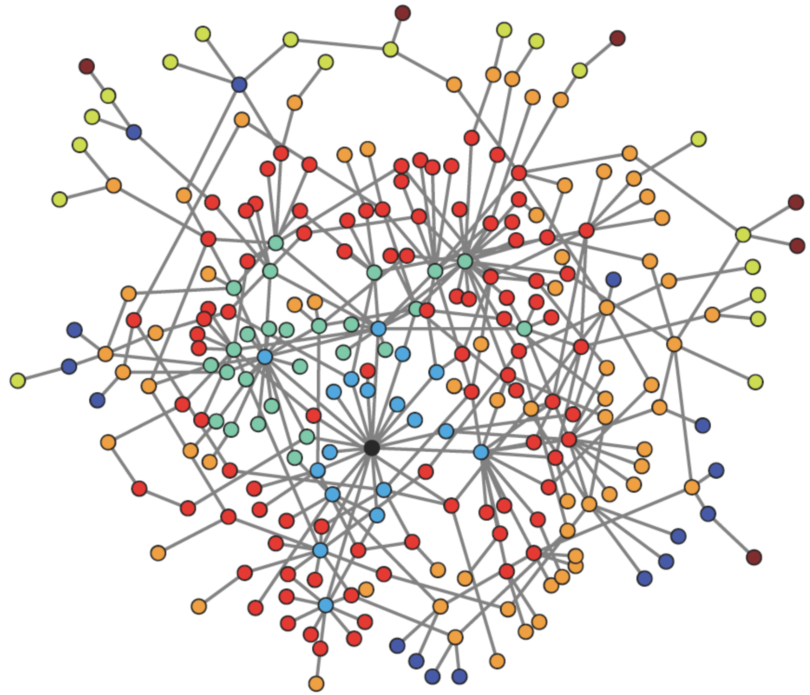
Results

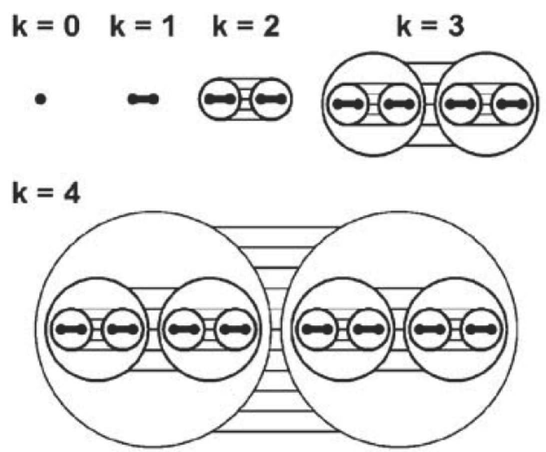




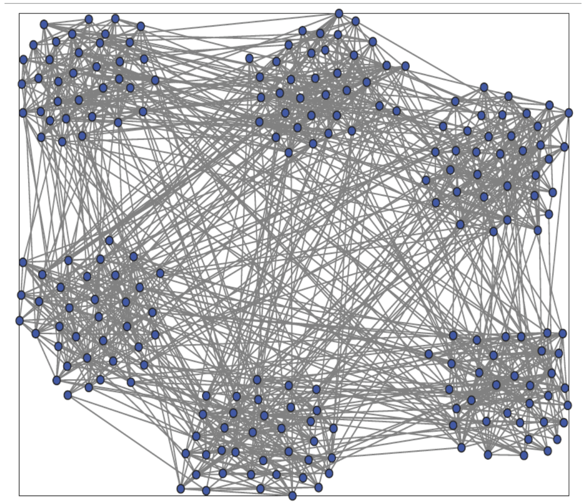
Müller-Linow et al., PLoS Comp. Biol. (2008) 4, e1000190

Excitable dynamics on graphs

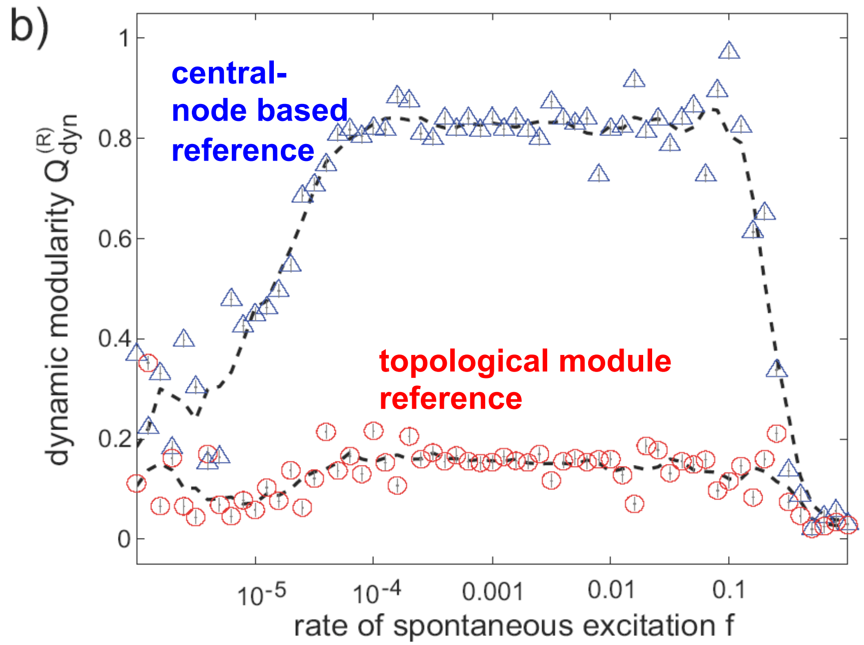
Application to models of biological neural networks

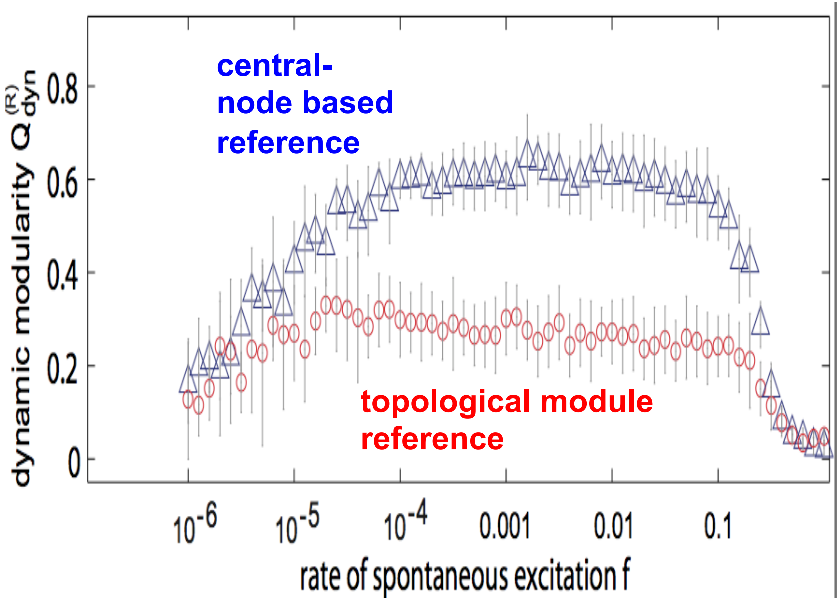


"fractal" graph taken from:
Sporns (2006) BioSystems 85, 55



hierarchical graph taken from:
Kaiser and Hilgetag (2007) Neurocomputing 70, 1829

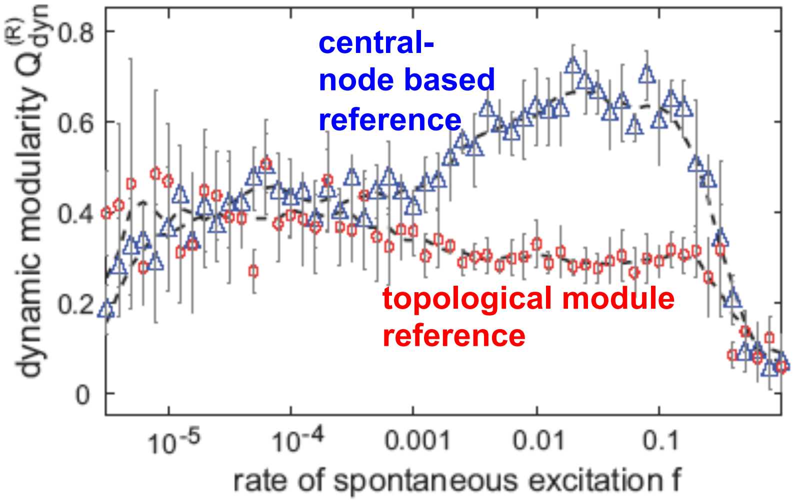




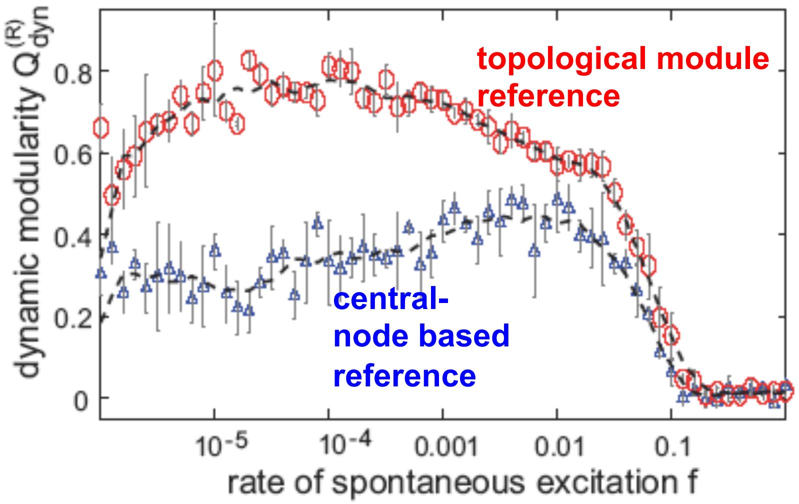
Excitable dynamics on graphs

Application to two real networks

cortical systems network of the cat
55 cortical areas; 565 connections

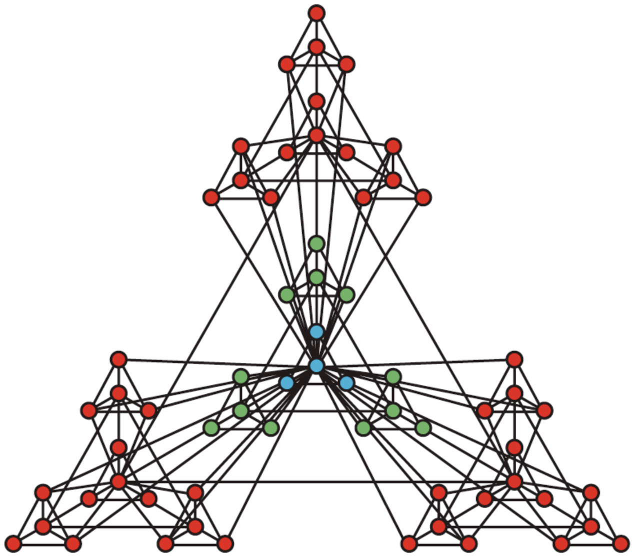


cellular neuronal network of *C. elegans*
277 neurons; 1918 connections



Excitable dynamics on graphs

Results (cont'd)



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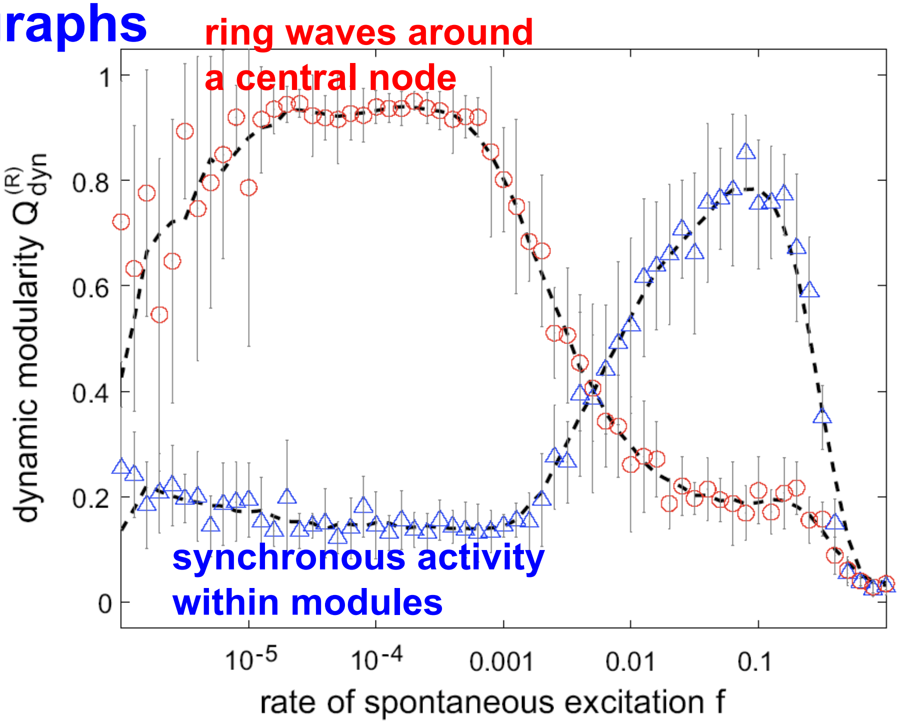
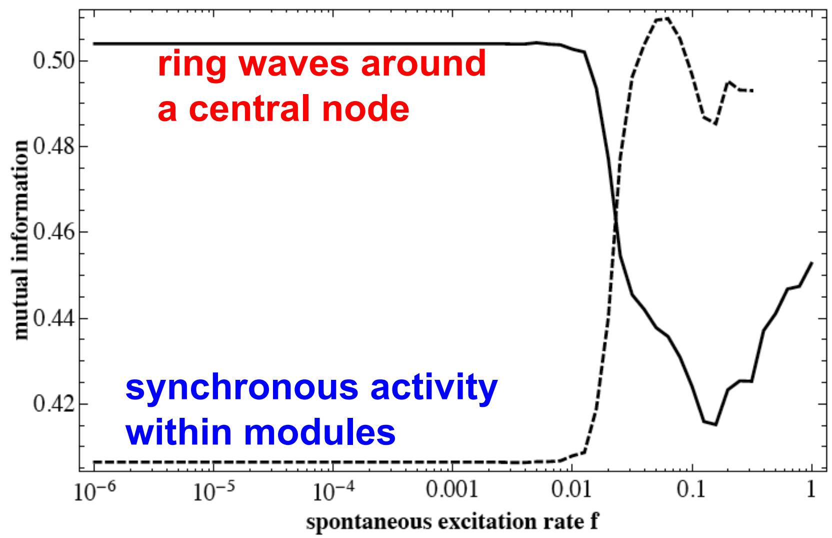
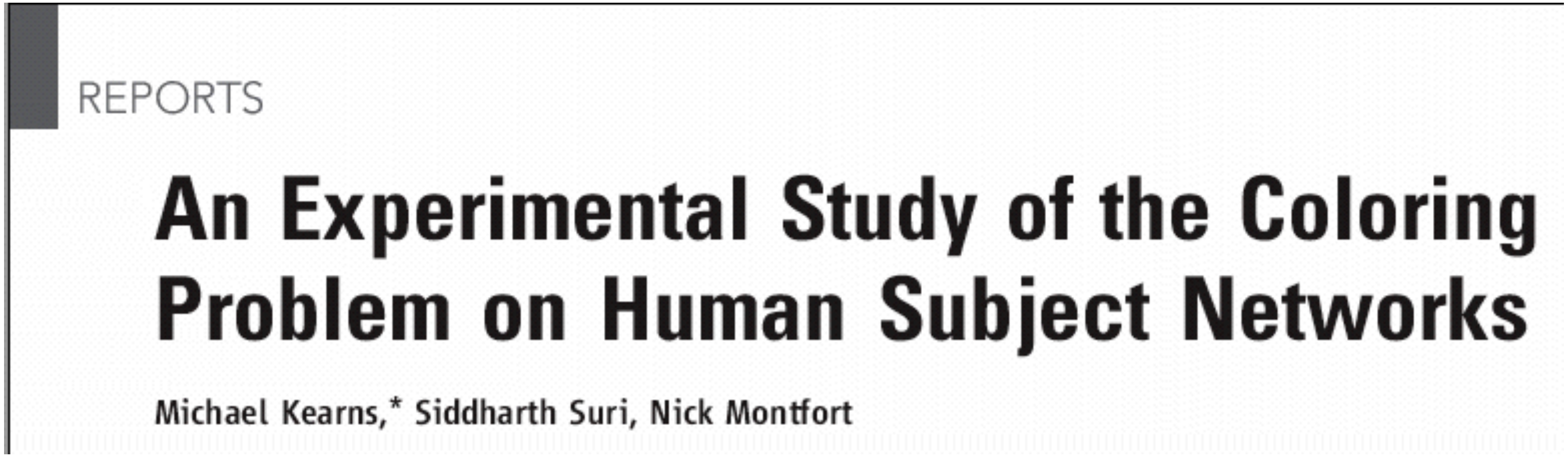




Hütt and Lesne (2009) Frontiers in Neuroinformatics 3, 28.

Graph coloring dynamics as a minimal model of collective problem-solving

Background


REPORTS

An Experimental Study of the Coloring Problem on Human Subject Networks

Michael Kearns,* Siddharth Suri, Nick Montfort


11 AUGUST 2006 VOL 313 SCIENCE www.sciencemag.org

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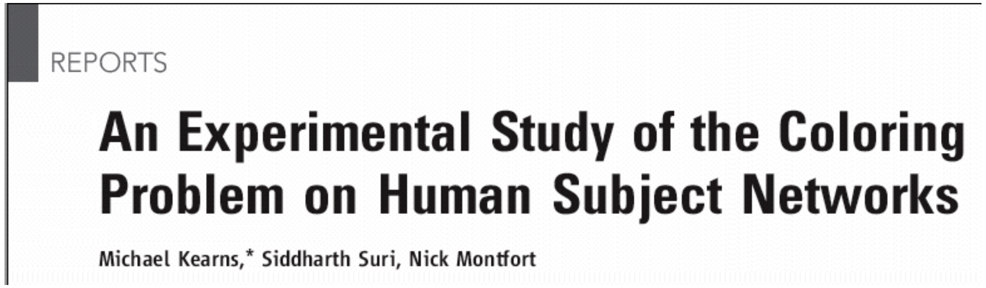
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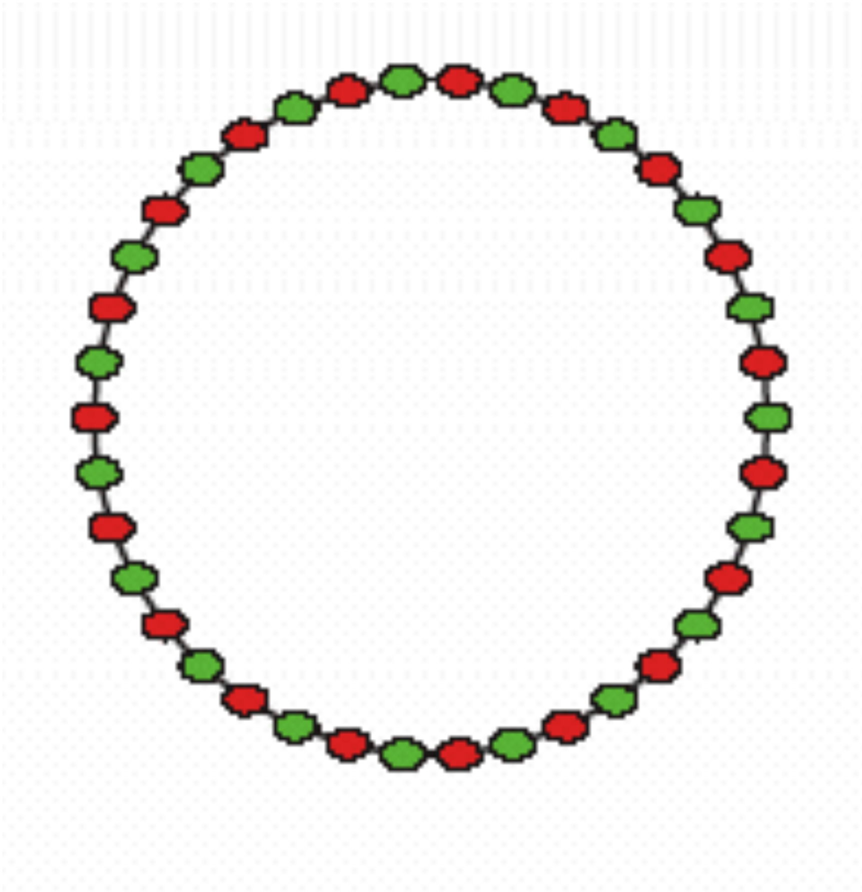

Taken from: Kearns et al. (2006) Science 313, 824

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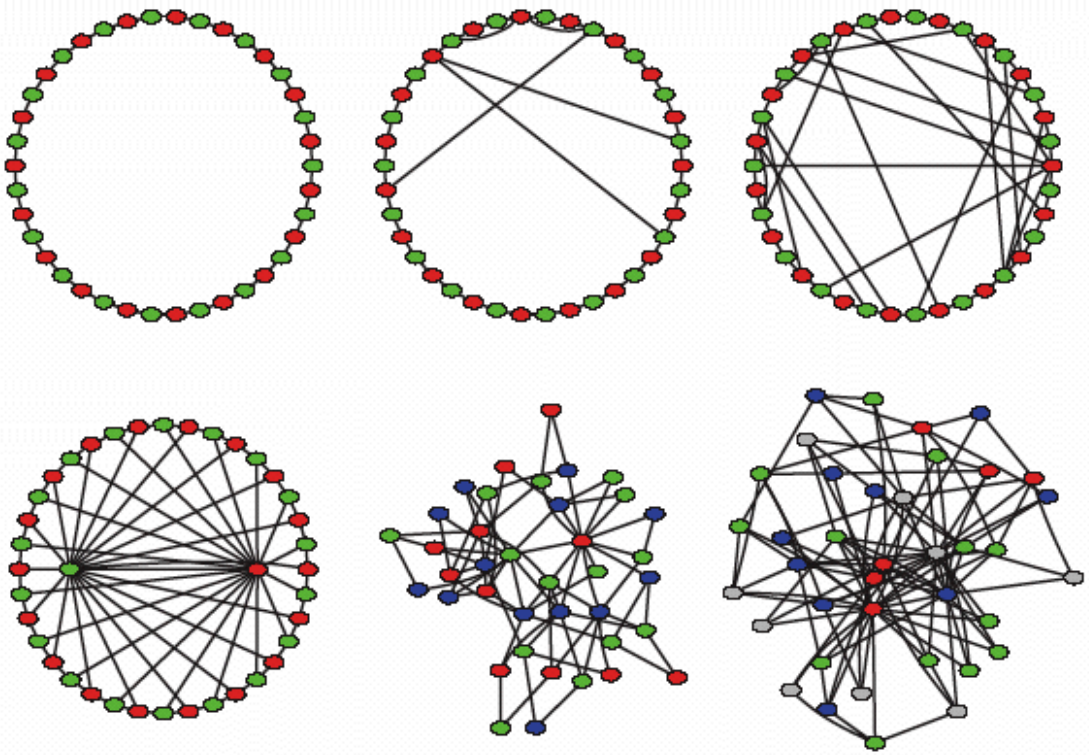

Fig. 1. Network topologies with sample colorings found by subjects. From left to right and top to bottom: simple cycle, 5-chord cycle, 20-chord cycle, leader cycle, and preferential attachment with two and three links initially added to each new vertex.

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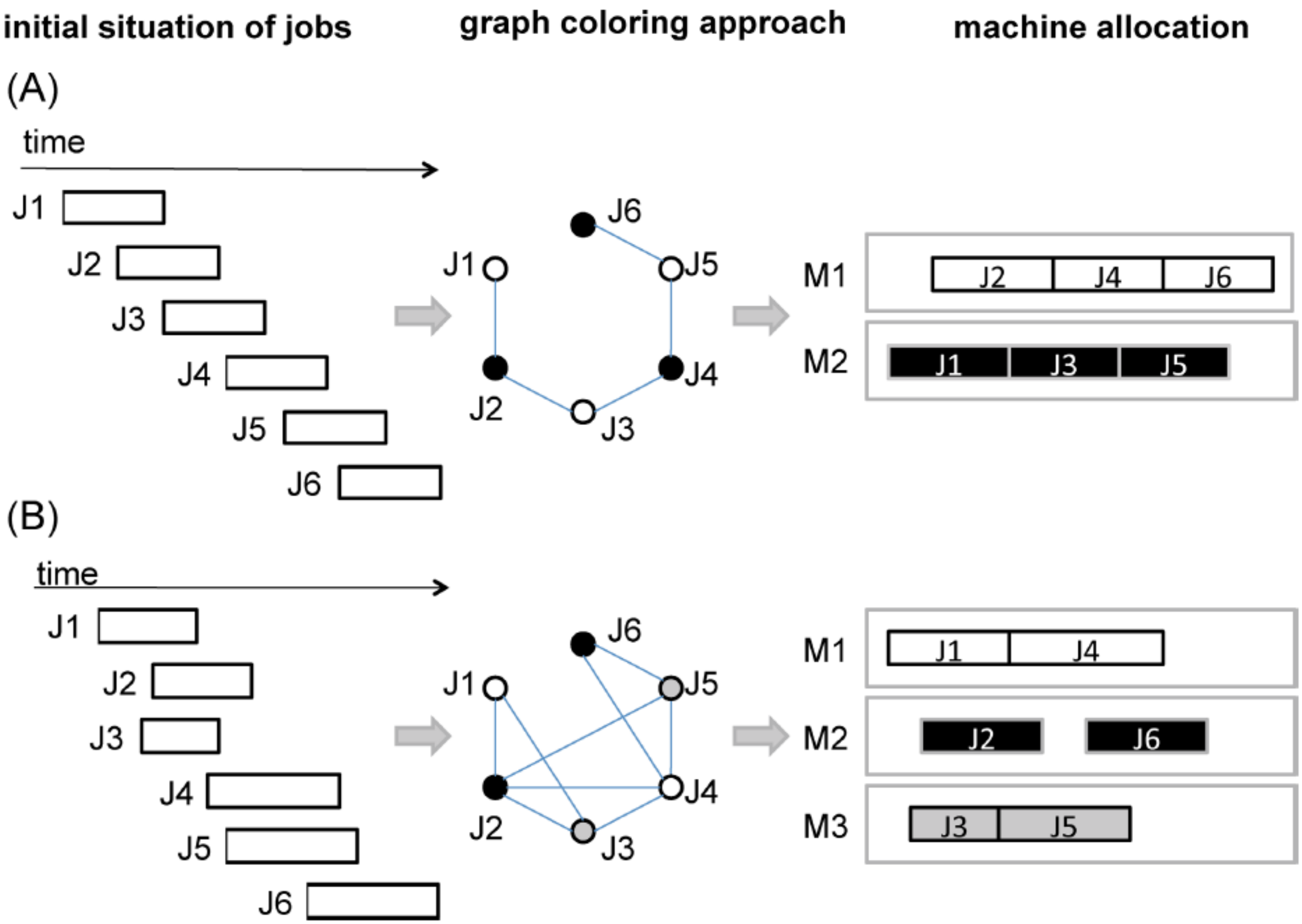

Windt and Hütt (2010) CIRP Annals Manufacturing Technology 59, 461.

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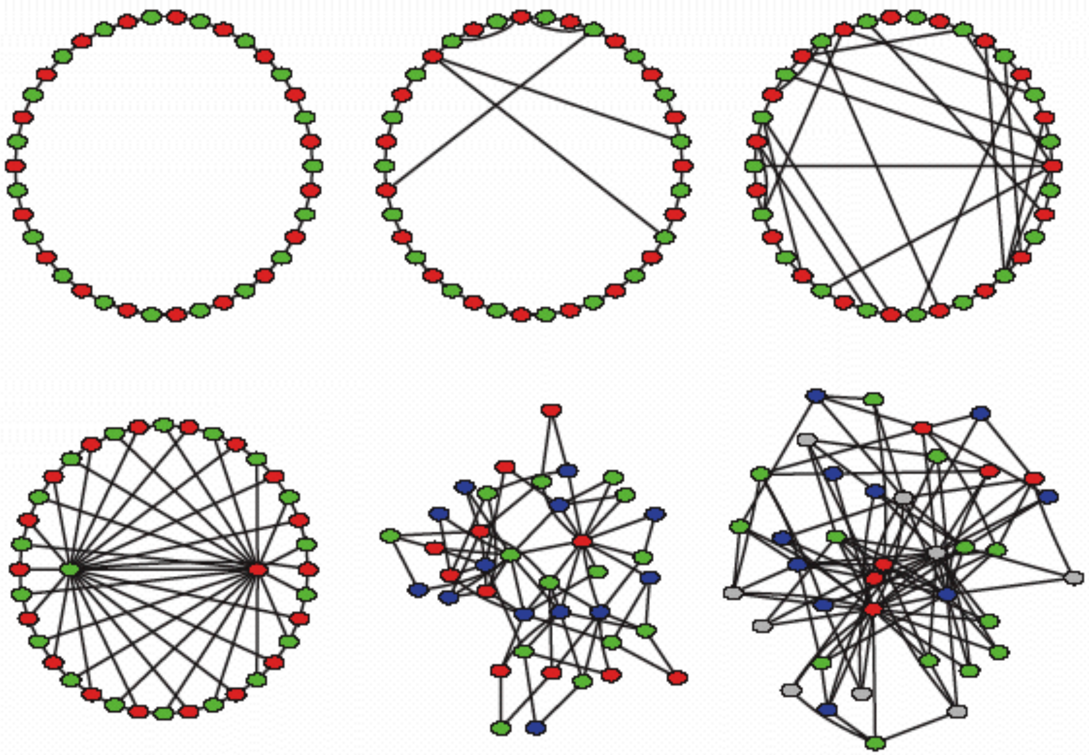

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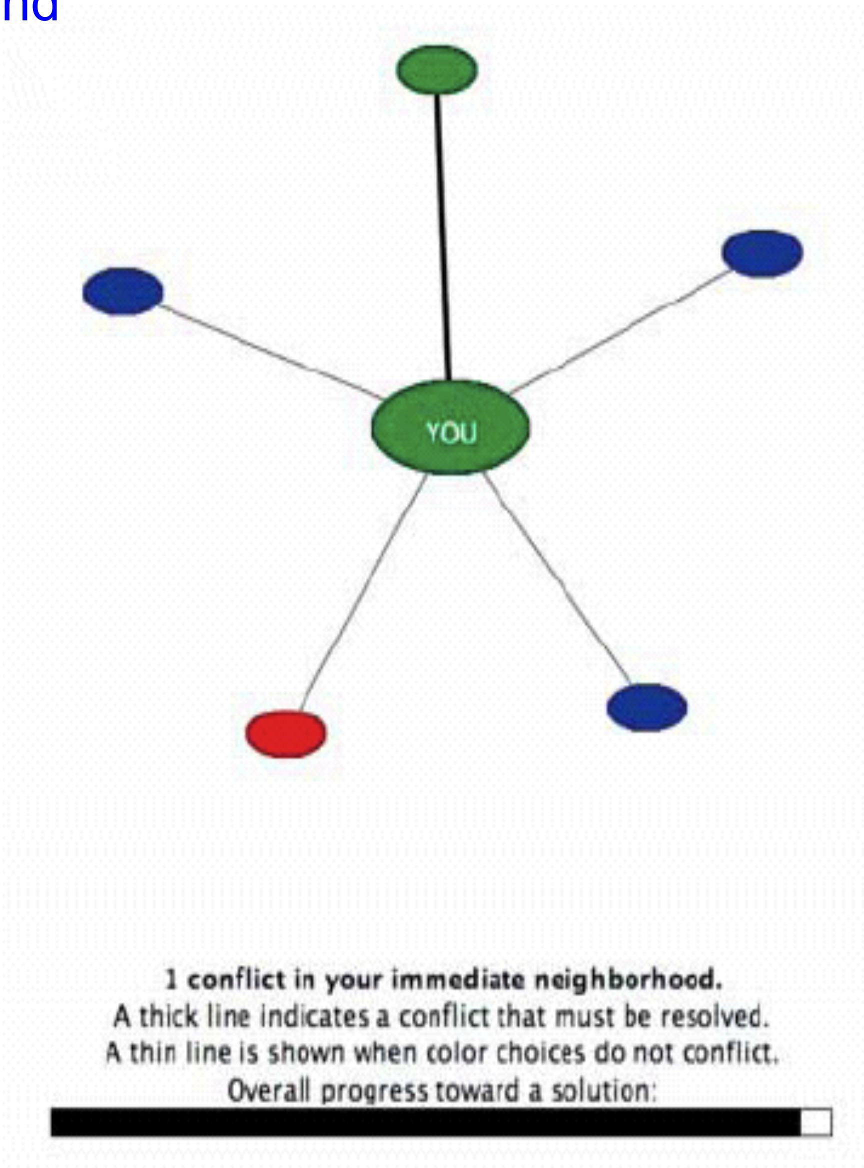

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Table 1. For each of the six experimental networks, the first six columns provide statistics summarizing structural properties, including the chromatic number (smallest number of colors required for solution), and statistics on the distribution of the degree (number of links) of each vertex. Network average distance is the average shortest-path distance, measured in number of links traveled, over all pairs of vertices. Also displayed are the average experiment duration for each network, along with the fraction of trials on which it was solved within 300 s and the number of steps (measured in color changes) for a natural distributed computer heuristic. Pref. att., preferential attachment.

	Graph statistics								
	Colors required (No.)	Min. links (No.)	Max. links (No.)	Avg. links (No.)	SD	Avg. distance (No. of links)	Avg. experiment duration (s) and fraction solved		Distributed heuristic (No. of color changes)
Simple cycle	2	2	2	2	0	9.76	144.17	5/6	378
5-chord cycle	2	2	4	2.26	0.60	5.63	121.14	7/7	687
20-chord cycle	2	2	7	3.05	1.01	3.34	65.67	6/6	8265
Leader cycle	2	3	19	3.84	3.62	2.31	40.86	7/7	8797
Pref. att., v = 2	3	2	13	3.84	2.44	2.63	219.67	2/6	1744
Pref. att., v = 3	4	3	22	5.68	4.22	2.08	154.83	4/6	4703

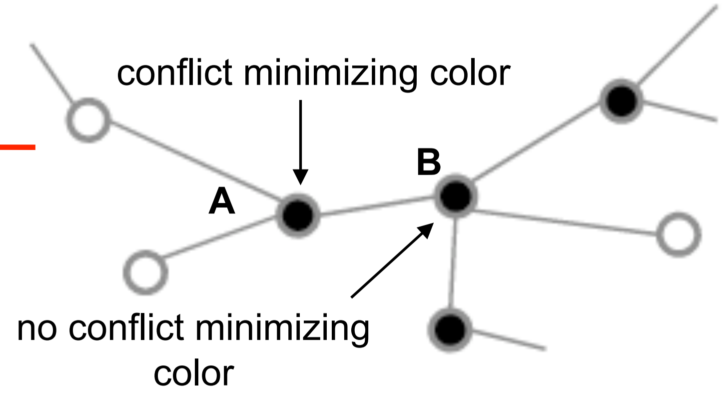
experiment **simulation**

Taken from: Kearns et al. (2006) Science 313, 824

Graph coloring dynamics as a minimal model of collective problem-solving

Our model

- attention waves: a color change in the neighborhood triggers an update of a node
- at each color change a node picks the color minimizing the number of conflicts
- strategic waiting: whenever the node is already in its conflict-minimizing color, with high probability it does not change its color

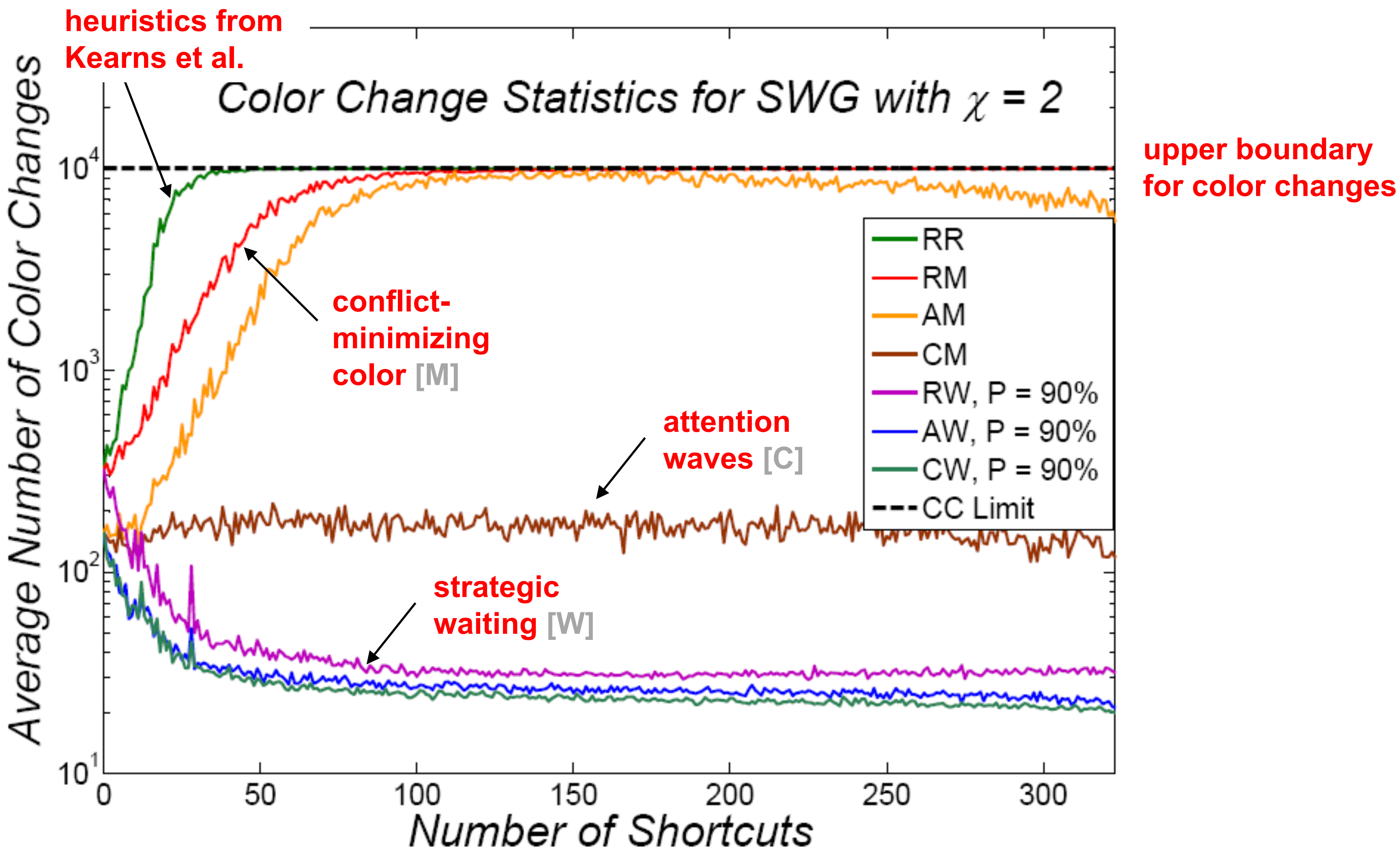


Strategic waiting of A since B is better placed to resolve the conflict

Hadzhiev et al. (2009) Advances in Complex Systems 12, 549.

Graph coloring dynamics as a minimal model of collective problem-solving

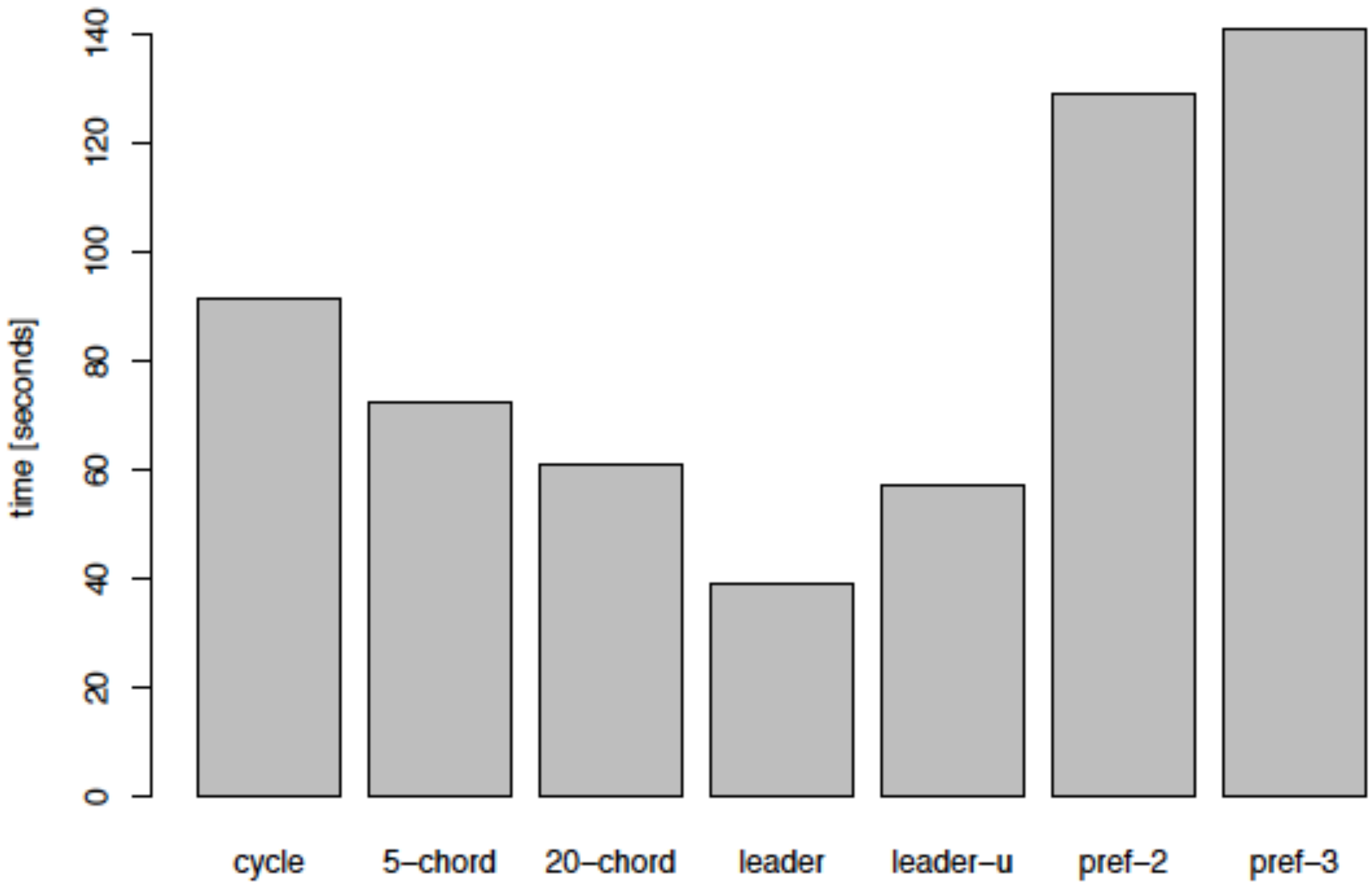
Results



Hadzhiev et al. (2009) Advances in Complex Systems 12, 549.

Graph coloring dynamics as a minimal model of collective problem-solving

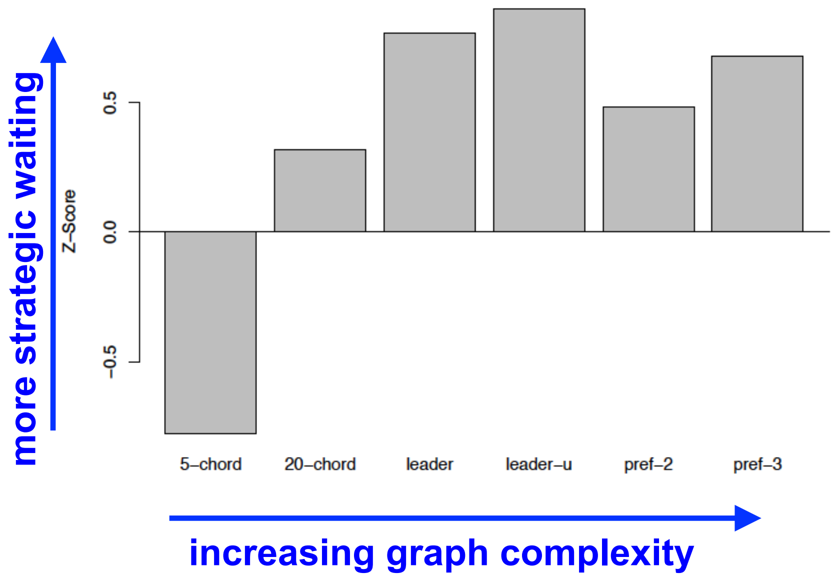
Results

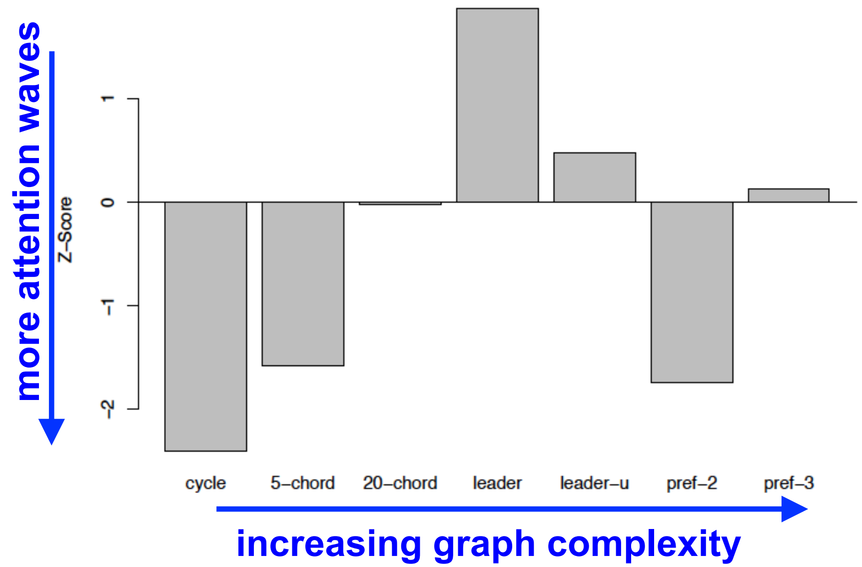


Kölling et al. (2011) EPJ B, *submitted*.

Graph coloring dynamics as a minimal model of collective problem-solving

Results

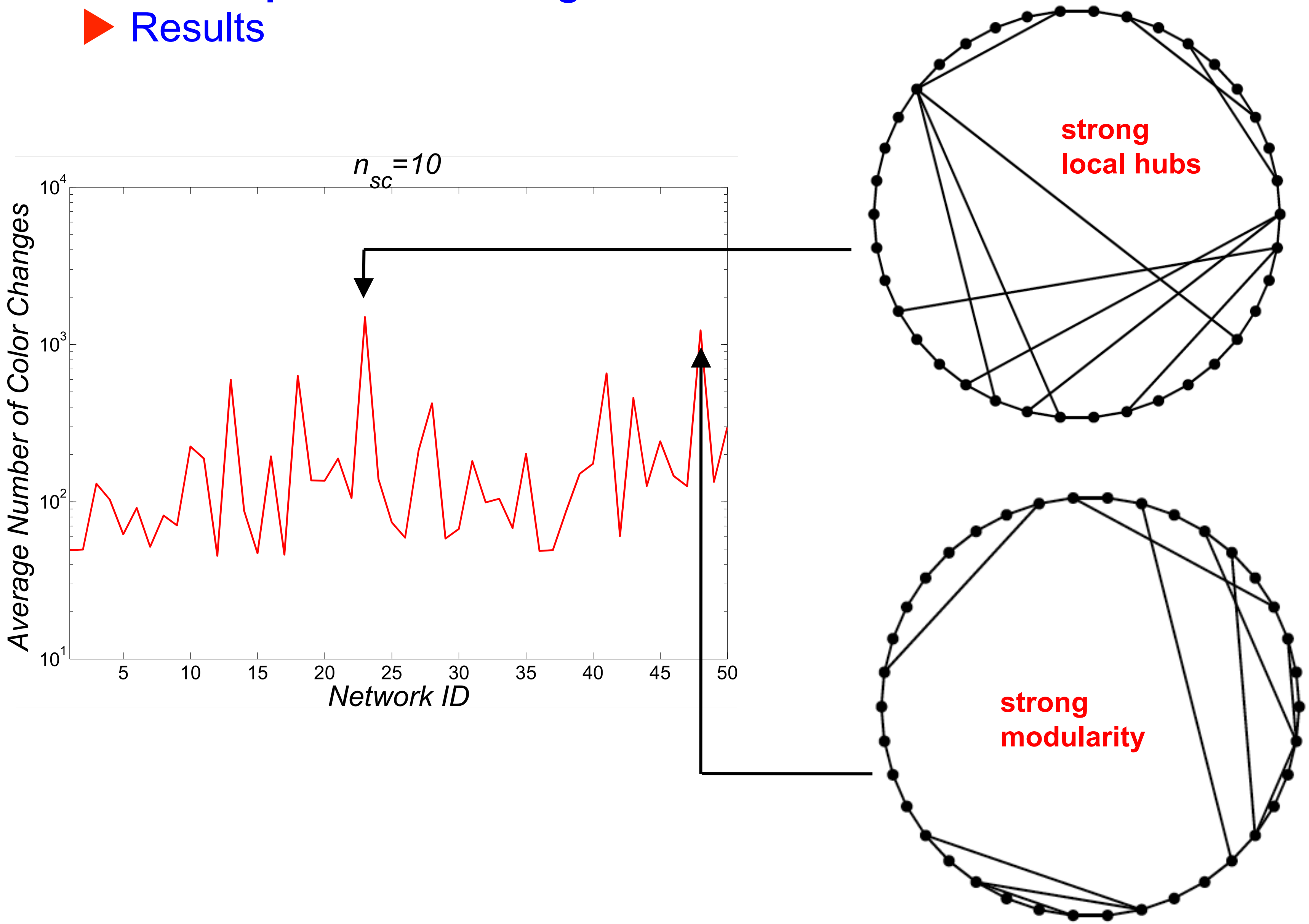




Kölling et al. (2011) EPJ B, *submitted*.

Graph coloring dynamics as a minimal model of collective problem-solving

Results



Hadzhiev et al. (2009) ***unpublished***

Summary

Three examples

(1) small case study on biological pattern formation

- individual properties of few elements can shape such patterns
- explicit pacemaker elements
- dynamically generated pacemakers

(2) excitable dynamics on graphs

- What are the network equivalents of spatiotemporal patterns?
- hubs as organizers of propagating waves
- synchronization of elements in the network, which are not necessarily linked

(3) an example of collective problem-solving

- graph coloring dynamics as a minimal model of a collective problem-solving task
- strategic waiting can help to resolve local deadlock situation
- exact placement of few shortcuts dramatically influences the performance

Acknowledgments

Contributors

Daniel Geberth, Miriam Grace **(pattern formation)**

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