Numerical experiments on intermittent linear response and spontaneous fluctuations in off-equilibrium aging dynamics

Paolo Sibani
IFK, Institut for Fysik og Kemi, SDU

Motivation

Simulation and experimental data

Record dynamics scenario

Summary & Conclusions

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Outline

1. Motivation
2. Simulation and experimental data
3. Record dynamics scenario
4. Summary & Conclusions

Background

- After a quench, complex *glassy* materials (glasses, polymers spin glasses) undergo a slow change of physical properties called aging[1] .

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- Averages (energy,magnetization etc) decay slowly (power-laws or logarithms)
- Lack of time translational invariance: the dynamics is never stationary.

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Biological examples

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In systems of biological interest the same type of non-stationary dynamics is also encountered

- Paleontological record (PS et al, PRL 1995, 1997)

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- Social dynamics of ant populations (H.J. Jensen, personal communication, 2008)

Macroscopic vs. mesoscopic

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- Mesoscopic systems have measurable fluctuations.
- Fluctuation spectra contain important dynamical information.

Why is aging interesting

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- Similar dynamical properties found across a range of complex systems motivate the search for simple unifying principles

1) Bissig et al. 2003, Cipelletti & Laurence 2005

2) PS & H J Jensen 2005.

Why is aging interesting

- Similar dynamical properties found across a range of complex systems motivate the search for simple unifying principles
- Experimental[1] and numerical[2] probes of *mesoscopic* systems link aging to intermittency & may provide detailed insights in the dynamical mechanism behind aging.

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2) PS & H J Jensen 2005.

This talk

- mainly describes intermittent fluctuations of

  Heat flow PDF
  Spontaneous magnetic fluctuations
  Linear-response functions

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- *record dynamics*• and *subordination* of aging to intermittency.

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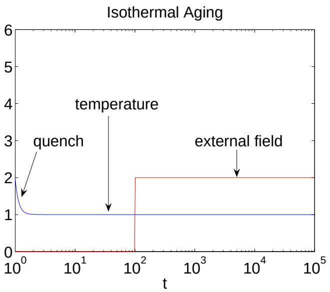
Aging set-up



The age t starts at the 'initial quench', and a field is (possibly) switched on at $t = t_w = 100$.

Questions:

How do ‘observables’ (=averages) depend on time(s)?

- Average energy depends on t

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- Average energy depends on t
- Average magnetization depends on t_w and t
- Correlation depends on t_w and t
- Spontaneous and induced fluctuation spectra depend on t_w and t

Edwards Anderson spin glass

The Edwards Anderson spin-glass is an Ising spin model where interacting spins $\sigma_i = \pm 1$ are placed on a lattice. The Hamiltonian is

- $\mathcal{H} = \sum_{n.n.} \sigma_i \sigma_j J_{ij}$.

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- For nearest neighbors J_{ij} are Gaussian standard random variables, independent for $i < j$
- The thermal equilibrium properties of the model are *complex*
- The time evolution, as given by a MC algorithm (Metropolis acceptance rule or equivalent) is *complex* and very similar to experimental data

p-spin glass model

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- $\mathcal{H} = -\sum_{Plaq.} \sigma_i \sigma_j \sigma_k \sigma_l$
- The thermal equilibrium properties of the model are *trivial*
- The time evolution is nevertheless complex, featuring metastability and aging.

ROM model

The Restricted Occupancy Model is a lattice model describing vortex creep in type II superconductors in terms of the number $n_i =$ of vortices on site i.

- the energy includes repulsive interactions between vortex lines on neighbor sites

see L. P. Oliveira et al. *Phys. Rev. B* 104526 (2005) and references therein.

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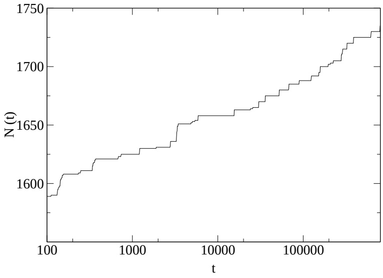
- the energy includes repulsive interactions between vortex lines on neighbor sites
- pinning to random sites
- the configuration is updated with Metropolis dynamics

see L. P. Oliveira et al. *Phys. Rev. B* 104526 (2005) and references therein.

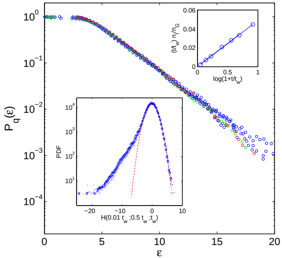
ROM model intermittency



The time variation of the total number of vortices $N(t)$ on the system for a single realization of the pinning potential and the thermal noise in a $8 \times 8 \times 8$ lattice for $T = 0.1$.

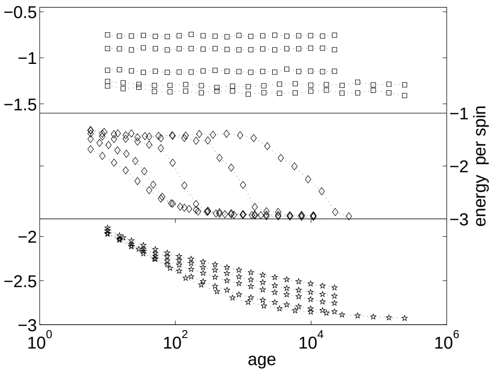
L. P. Oliveira et al. Phys. Rev. B 104526 (2005)

Spin-glass, heat flow intermittency



Heat transfer PDF for a spin glass model (PS & H J Jensen, Europhysics Lett. 2005) Heat transfer H over small time δt in the E-A spin glass model has a Gaussian part and an intermittent tail. Six different ages are considered with $\delta t/t_w = .01$.

p-spin model, average energy decay

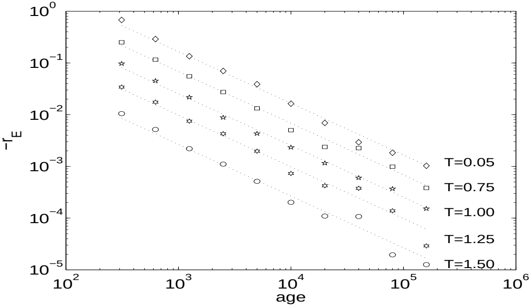


PS, *Phys. Rev. E*, 74:031115, 2006

Lipowski & Johnson, *Phys. Rev. E*, 61:6375–6382, 2000

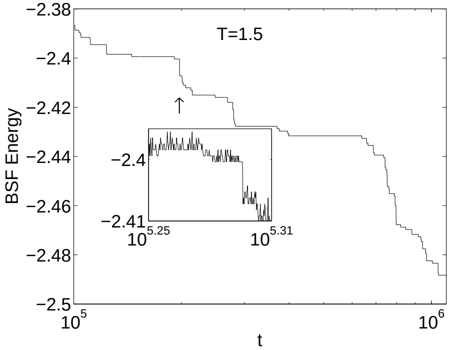
Swift et al. *Phys. Rev. B*, 62:11494–11498, 2000.

Heat flow rate, p-spin model



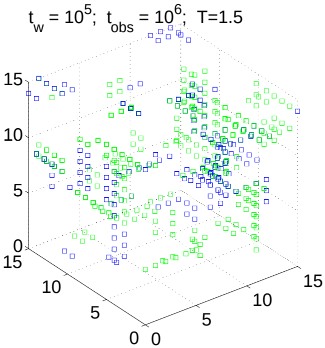
The average rate of energy flow is plotted versus the age for the temperatures shown. The full line has the form $y = C(T)t_w^{-1}$.

p-spin model, intermittent energy decay



S. Christiansen & PS, *New J. of Physics*, to appear

p-spin model, quakes in real space



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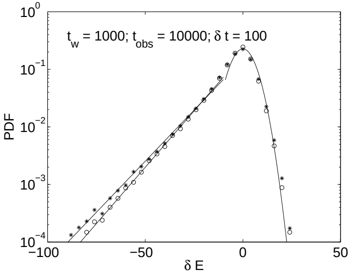
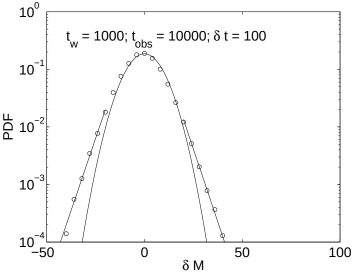
p-spin model, heat flow



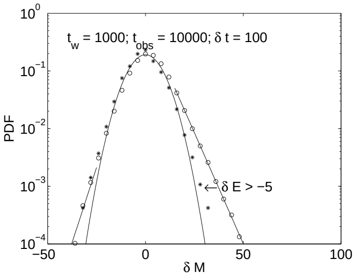
The PDF of the heat exchanged between system and thermal bath over a time $\delta t = 100$. $T = 1.5$.

p-spin model, magnetic fluctuations $H = 0$



The PDF of the spontaneous magnetic fluctuations over a time $\delta t = 100$ and $T = 1.5$.

p-spin model, magnetic fluctuations $H = 0.3$



The PDF of the spontaneous magnetic fluctuations over a time $\delta t = 100$. $T = 1.5$.

The magnetic response is SUBORDINATED to the quakes

E-A spin-glass, autocorrelation function

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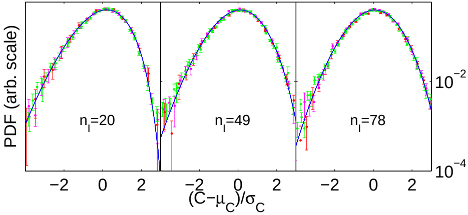
The auto-correlation measures configurational changes occurring while the system ages

Definition: $C(t_w, t) = \frac{1}{N}\sum_i \sigma_i(t_w)\sigma_i(t + t_w)$.

For finite N, this is fluctuating quantity.

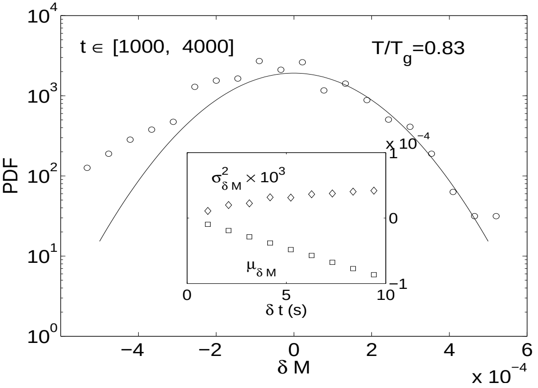
The distribution of the fluctuations carries informations on the configurational changes induced by the quakes.

E-A spin-glass, autocorrelation fluctuations



Scaled and shifted autocorrelation PDF (full line)& simulation data for $T = 0.15, 0.25, 0.35$ and 0.5. $t/t_w = 2.3, 7.6$ and 25.6,left to right. PS, Europh. Lett.73, 2006.

Spin-glass TRM flow



The PDF of the magnetization change δM for isothermal aging at $T = 0.83\,T_g$. The magnetic field is cut at $t_w = 100$s. The sampling interval is $[1000, 3000]$ and $\delta t = 5 \times 1.045$s. PS, G. Kenning and G. Rodriguez, *Phys. Rev. B* 74, 224407 (2006)

Two types of events

Two types of changes characterize the evolution of aggregate variables (i.e. the energy)

- Reversible fluctuation of zero average with Gaussian PDF's

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- Reversible fluctuation of zero average with Gaussian PDF's
- Irreversible events quakes which carry the aging drift
- & produce *intermittent* tails—usually exponentials.

Subordination of aging phenomena to the quakes

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- The quakes control the aging process:
- correlations and response functions and other averages *only* depend of the number n of quakes occurring in the observation interval.
- The de-correlation effect of equilibrium-like fluctuations is neglected in this *approximation*.

Temporal statistics of quakes

- The temporal statistics of quakes can be studied empirically from signal traces

Temporal statistics of quakes

- The temporal statistics of quakes can be studied empirically from signal traces
- if one is able to identify the times $t_1, t_2 \ldots t_k$ at which the quakes 'occur'.

E.g. the p-spin model-again

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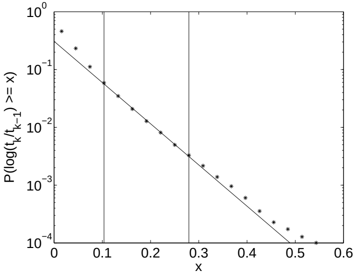

The distribution of the logarithmic differences $\log(t_k) - \log(t_{k-1})$. is approximately exponential.

The ‘log’ Poisson distribution

- $P(n, t_1, t_2)$ probability that n quakes occur in $[t_1, t_2)$.

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- $P(n, t_1, t_2)$ probability that n quakes occur in $[t_1, t_2)$.
- $$P(n, t_1, t_2) = \frac{\mu^n}{n!} \exp(-\mu) \quad \mu(t_1, t_2) = \alpha \log(t_2/t_1) \quad (1)$$

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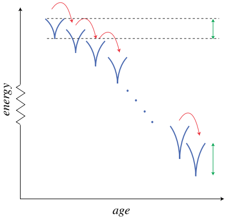
- The statistics applies in MANY situations
- Where could it possibly come from?

Metastability & marginal stability

- Metastable attractors = attractor basins of noise-less dynamics
- Reversible, equilibrium-like fluctuations within the basins
- Large intermittent fluctuations correspond to a change of attractor
- The nature of the noise is system specific
- What would trigger a change of attractor in isothermal aging?

Extremal (record) fluctuations



- Extremal fluctuations are irrelevant in stable systems
- But can bring a metastable system 'over the stability edge' of a basin
- In a *marginally stable* situation, each extremal fluctuation does so

'log-Poisson' distribution from record statistics

Consider a time series of independent random numbers with any continuous distribution.
A record is a number larger than all its predecessors.

- The number of records between draws t_1 and $t_2 > t_1$ is well described as a Poisson process

(P.S & Peter Littlewood PRL 1993)

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Consider a time series of independent random numbers with any continuous distribution.

A record is a number larger than all its predecessors.

- The number of records between draws t_1 and $t_2 > t_1$ is well described as a Poisson process
- with average $\log(t_2/t_1)$

(P.S & Peter Littlewood PRL 1993)

Method of analysis

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- Solve a Markov chain where n has the role of a 'time' variable. Leads to exponential decays.

Method of analysis

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- Average over the probability that n events occur within a time interval $(t_w, t_w + t)$ This is a Poisson distribution whose average scales with $\log((t_w + t)/t_w)$.
  The average over n turns the exponential into power-laws.

Minimal model of quake dynamics

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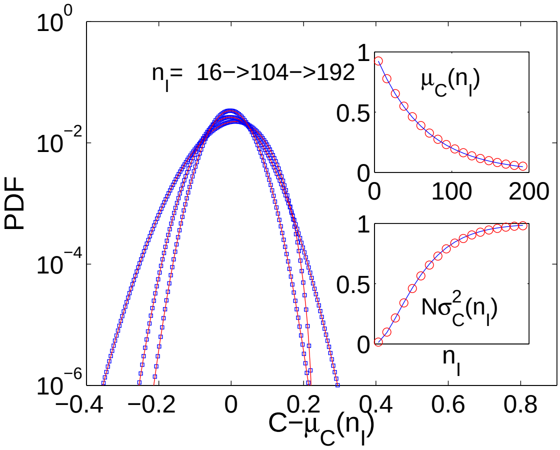
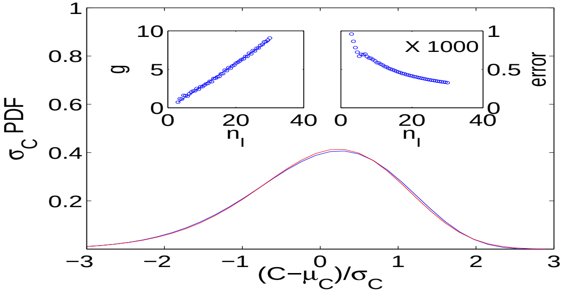
- All spins have the same probability to be part of a quake
- The number of quakes falling in certain time interval is ‘log’Poisson distributed.

Autocorrelation model PDF



Evolution of the model autocorrelation PDF. PS, Europh. Lett.73, 2006.

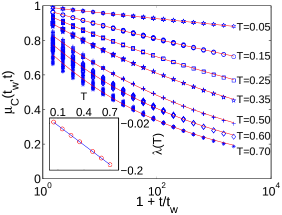
Comparison with Gumbel description



PDF is compared to the best Gumbel approximation.

The model see also: H. E. Castillo et al., Phys. Rev. B, 68:13442, 2003

Average autocorrelation

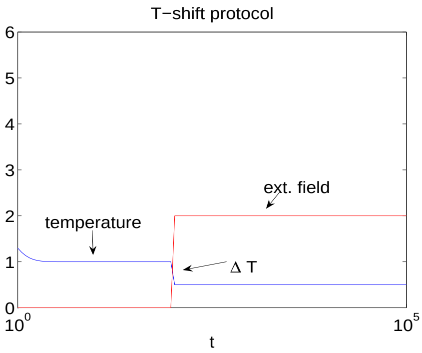


$$\langle C \rangle = (1 + t/t_w)^{\lambda(T)};$$

and

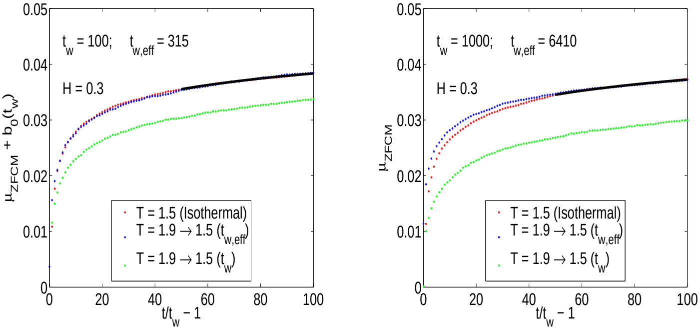
$$\lambda(T) = -2\frac{\alpha(N)}{N}\frac{1}{q(T)}$$

T-shifts



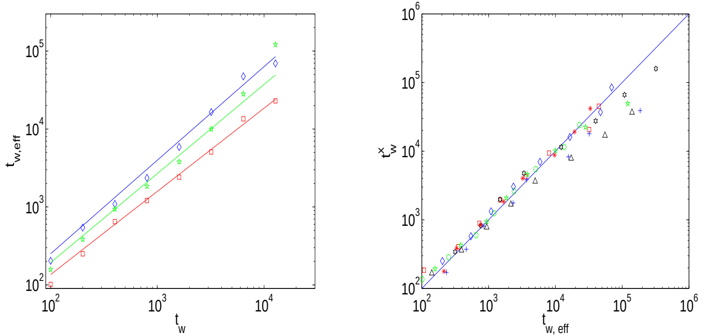
A negatve T-shift at $t = 100$

Average response & effective age



The average linear response (under isothermal or T-shifted conditions) versus the system age t, scaled with either the true or the effective age. The two nearly overlapping data sets are for: *i)* Isothermal response at the indicated temperature, versus the age scaled by t_{w}. *ii)* T-shifted response, plotted versus the system age scaled by $t_{w,\mathrm{eff}}$.(PS & Simon Christensen, PRE 77, 041106, 2008)

Effective age vs age & effective age



Left: algebraic relation between the effective and the actual age for the three different T-shifts Full lines: $t_{\mathrm{w,eff}} = t_{\mathrm{w}}^{x}$, where x is determined by a least square fit. Right, symbols t_{w}^{x} versus $t_{\mathrm{w,eff}}$ for all available x value. Line prediction $t_{\mathrm{w,eff}} = t_{\mathrm{w}}^{x}$ with $x = \frac{T_{\mathrm{initial}}}{T_{\mathrm{final}}}$. (PS & S. Christensen, PRE 77, 041106, 2008)

Data from mesoscopic systems

- experimental and numerical evidence from meso-scaled complex materials

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- for two types of events

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- experimental and numerical evidence from meso-scaled complex materials
- for two types of events
- equilibrium-like fluctuations & intermittent quakes

Record-dynamics interpretation

- the quakes are irreversible

Record-dynamics interpretation

- the quakes are irreversible
- are triggered by extremal fluctuations within IS basins

Configuration & real space properties

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- Similar energy landscapes belong to each of $\alpha(N)$ independently thermalized domains
- Quakes are not scale invariant objects in real space.
- Complex behavior arises due to a self-similar structure of the energy landscape of each domain.

Outlook

- the record dynamics scenario also apples to other non-thermal systems

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- The Tangled Nature model of evolution (based on interactions between individuals)

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