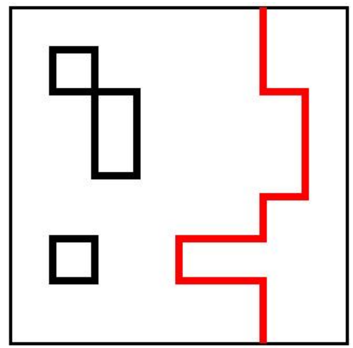
Ground state properties of the SOS model on a disordered substrate

dislocations and flat-to-superrough transition



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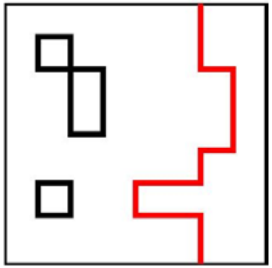
Content

- Introduction
- SOS Model with dislocations
- Algorithms: min-cost-flow, loop detection
- Results: transition, dislocations
- Summary & Outlook

Introduction

randomly pinned elastic medium models …

- crystal surface on disordered substrates [Toner et al. 1990]
- flux-line arrays in dirty superconductors [Blatter et al.1994]
- charge density waves (CDW) [Grüner1990]

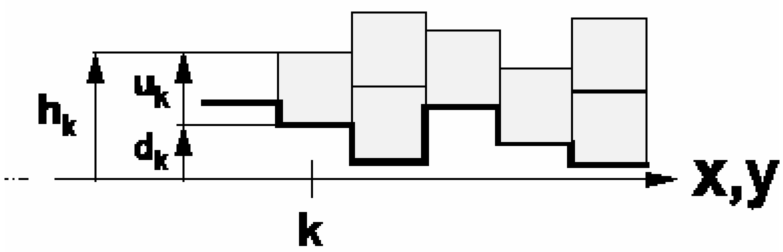


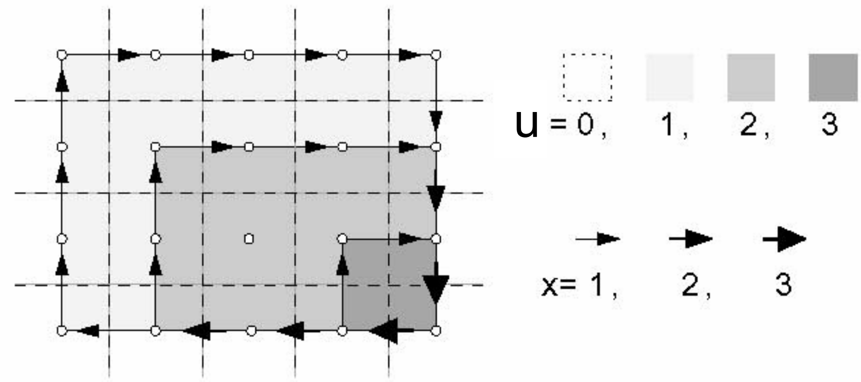
superrough-to-rough ($\log^2$-log) transition at T_c [Toner et al.1990]

at low T: randomness >> thermal fluc.

ground state:
superrough-to-flat transition at σ_c ?
dislocation proliferation (difficult for RG)?

Solid-on-Solid (SOS) model





$H_{SOS} = \sum_{\langle k,l \rangle} (h_k - h_l)^2$ $h_k = u_k + d_k$ **height-profile**

contour loops = lines of equal height: $\nabla \times \nabla u = x$ => $\nabla x = 0$

$H_{SOS} = \sum_{\langle k,l \rangle} (x_{kl} - b_{kl})^2$ **s. t.** $\nabla \cdot x_i = 0$ **contour profile**

height difference $x_{kl} = u_k - u_l \in$ integer

offset-difference $b_{kl} = d_k - d_l \in [-2\sigma, 2\sigma]$ uniform, uncorrelated

parameter: **disorder strength** $\sigma \in [0,1/2]$

Extreme cases at T=0

σ **= 0**: flat case

σ **= 1/2**: superrough, i.e.

$C(r) \sim \log^2(r)$

for $r \to \infty$

[Rieger et al. 1996]

calculation of **exact** ground state with **min-cost-flow algorithm** from combinatorial optimization

finite system size:

lattice propagator $C(r) \to P(r)$

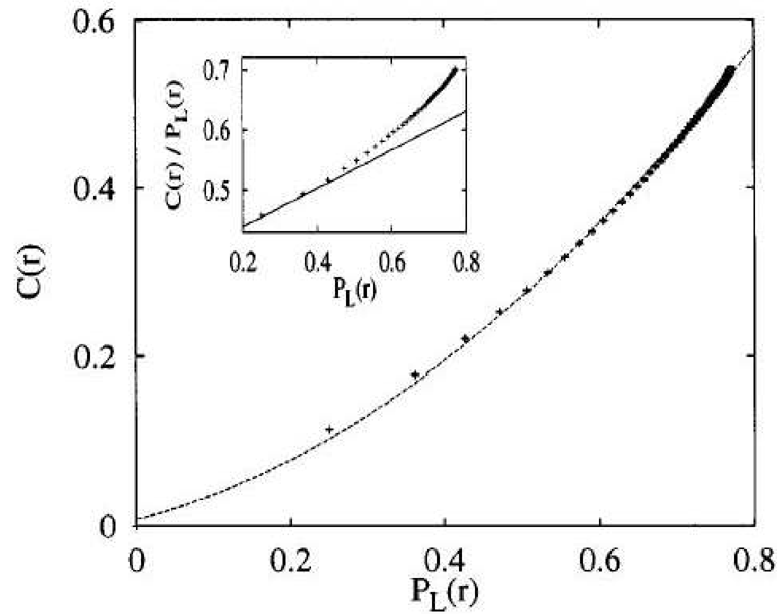
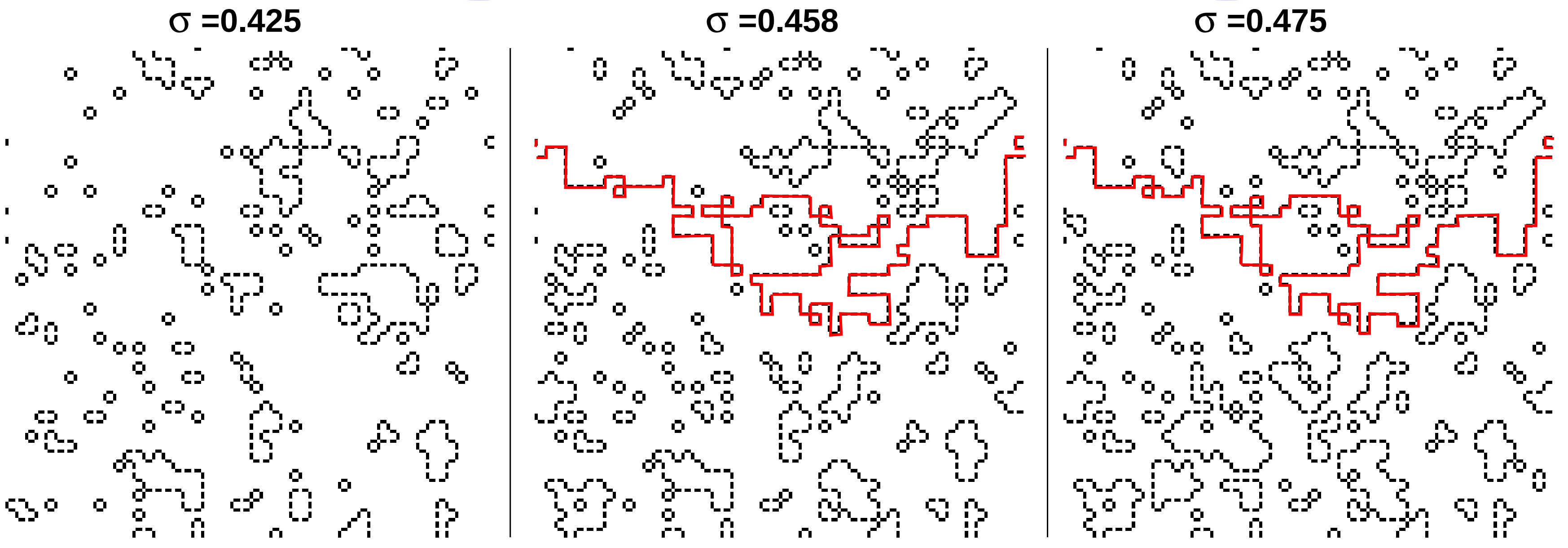


FIG. 1. The site averaged correlation function $\overline{C}(r)$ versus the lattice propagator $\overline{P}_L(r)$ for $L=128$ and averaged over 2000 samples. The broken line is a least square fit to $\overline{C}(r) = 0.008 + 0.21\overline{P}_L(r) + 0.57\overline{P}_L(r)^2$. The inset shows $\overline{C}(r)/\overline{P}_L(r)$ versus $\overline{P}_L(r)$, and the straight line indicates the amount of curvature of the data.

[Rieger et al. 1996]

disorder-driven phase transition

Percolation transition of contour loops



typical ground state configurations for increasing disorder strength σ

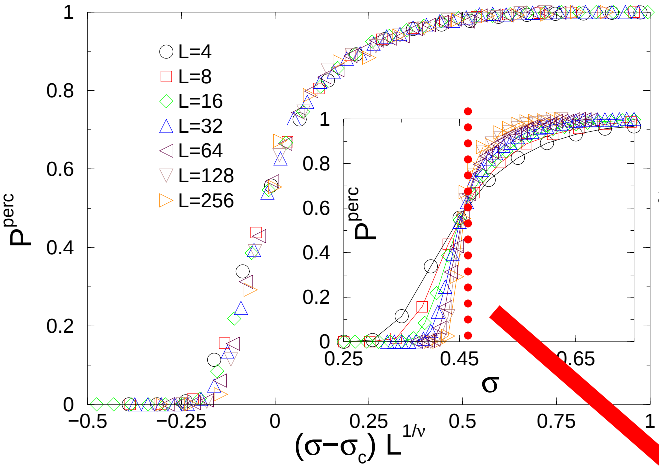
=> critical threshold $\sigma_c \approx 0.45$

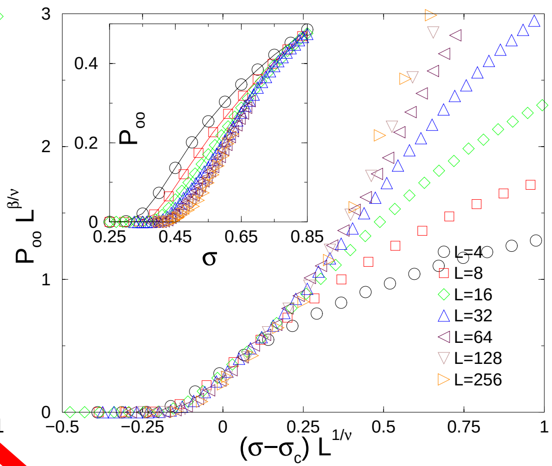
Loop detection algorithm

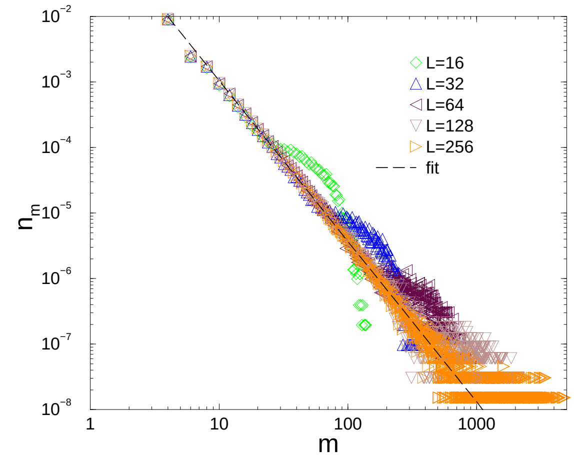
```
algorithm depth-first search along bonds;
begin
    create a loop configuration x(e) ∈ {0, ±1, ±2, ...}
    label(e) := 0   and   size(e) := 0   for all e ∈ E;
    t := 1;
    forall e ∈ E do
        if x(e) ≠ 0 and label(e) = 0 then
            depth-first( e );
            t = t + 1;
        endif;
    enddo;
end;
```

```
subroutine depth-first( e );
begin
    label(e) = t;
    size(e) = size(e) + |x(e)|;
    forall neighbors ẽ ∈ E of e do
        if x(ẽ) ≠ 0 and label(ẽ) = 0 then
            depth-first( ẽ );
        endif;
    enddo;
end;
```

Finite-Size Scaling







percolation probability P^{perco}

$\Rightarrow$ **critical threshold** $\sigma_c = 0.458 \pm 0.001$

$P^{perco} = P[\, L^{1/\nu}(\sigma-\sigma_c)\,]$ $\quad$ $P_\infty = L^{-\beta/\nu}\, P[L^{1/\nu}(\sigma-\sigma_c)]$ $\quad$ $n_m \sim m^{-\tau}$

ν	β	τ
3.33 ± 0.30	*1.80 ± 0.35*	*2.45 ± 0.05*

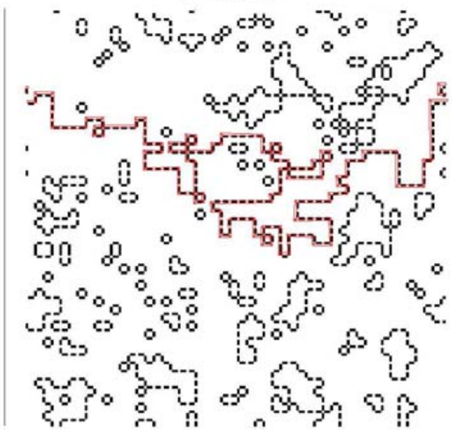
Universality class

geometrical exponents

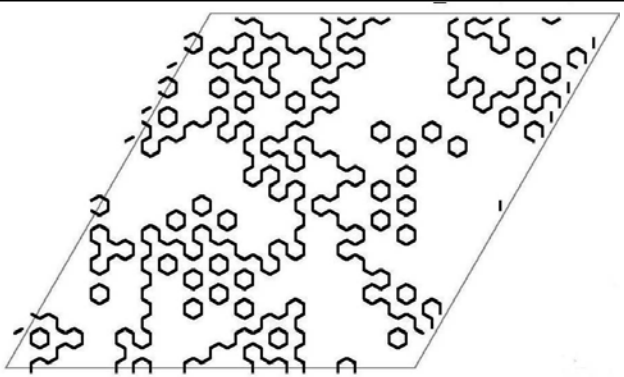
* in phase far from critical point

$$d_f = d - \beta/\nu$$

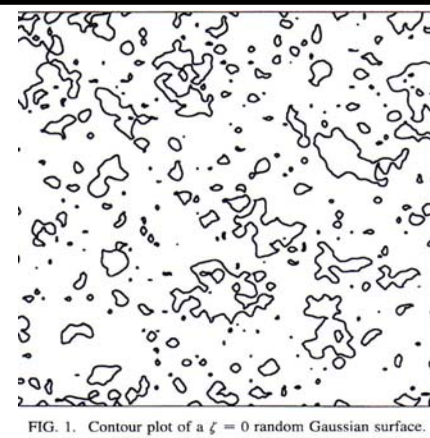
Model for d=2	d_f	τ
Solid-on-solid (SOS) at σ_c	**1.45±0.05**	**2.38±0.17**
Random elastic medium (REM)* [Zeng et al.1998]	1.46±0.01	2.32±0.01
Random Gaussian surface (RGS)* [Kondev et al. 1995]	1.49±0.01	2.35±0.03



SOS model at σ_c



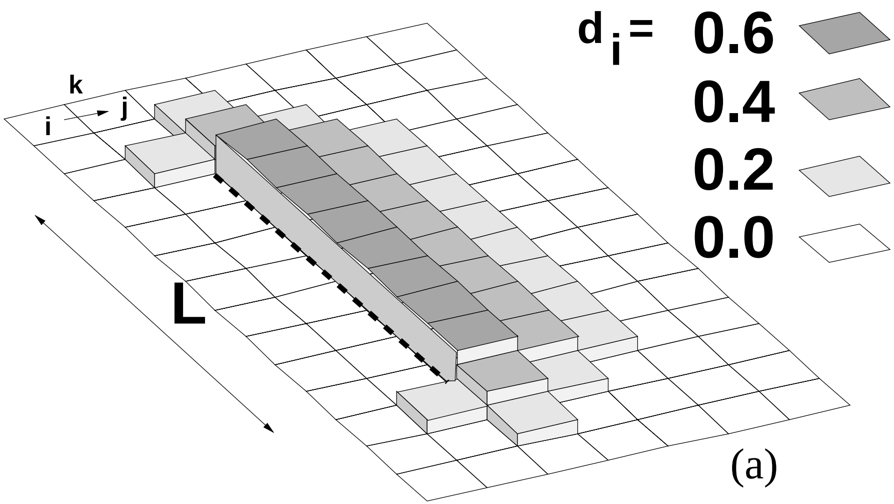
REM model with CL



RGS model

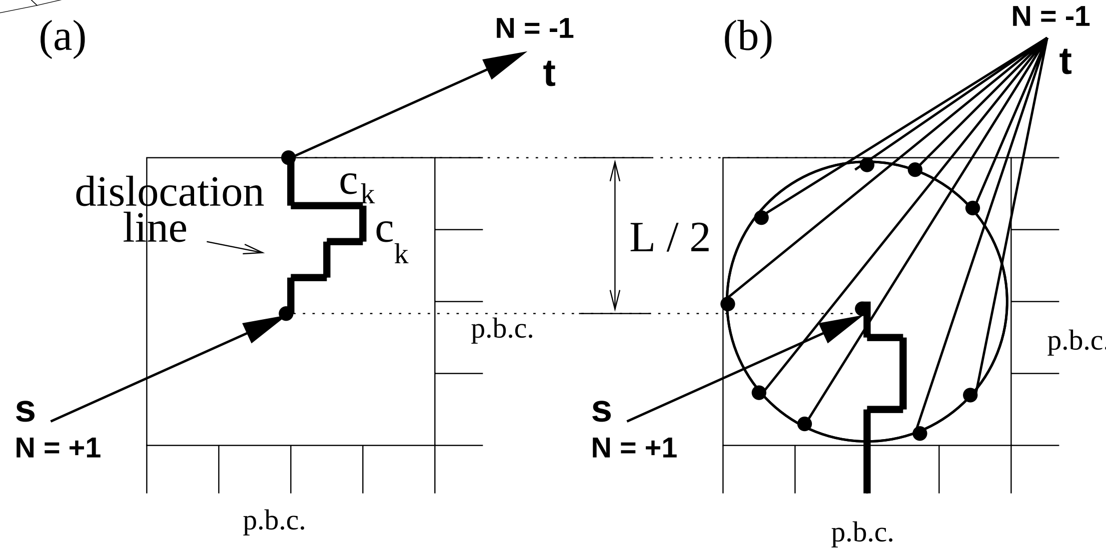
dislocations in superrough phase

Dislocations at σ = 1/2



example of disordered substrate with a single **dislocation pair**

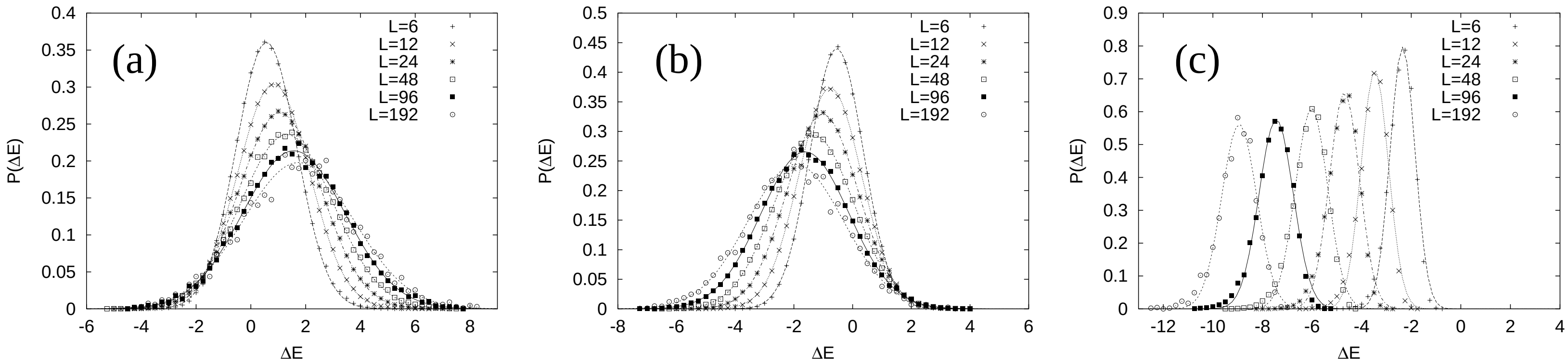
optimal configuration: n_i=0
dislocation => lower ground state



implementation
Lx**L** lattice with p.b.c.

1. fixed pair
2. partially opt. pair
3. completely opt. pair

Single defect pair (N=1)



fixed pair | partially opt. pair | completely opt. pair

defect energy

$$[\Delta E]_{\text{dis}} \sim \begin{cases} \ln(L) & \text{fixed defect pair} \\ -0.27(7) \times \ln^{3/2}(L) & \text{partially optimized} \\ -0.73(8) \times \ln^{3/2}(L) & \text{completely optimized} \end{cases}$$

fixed defect pair $\sim$ **$E_{el} \sim E_{el}^{pure}(T)$**

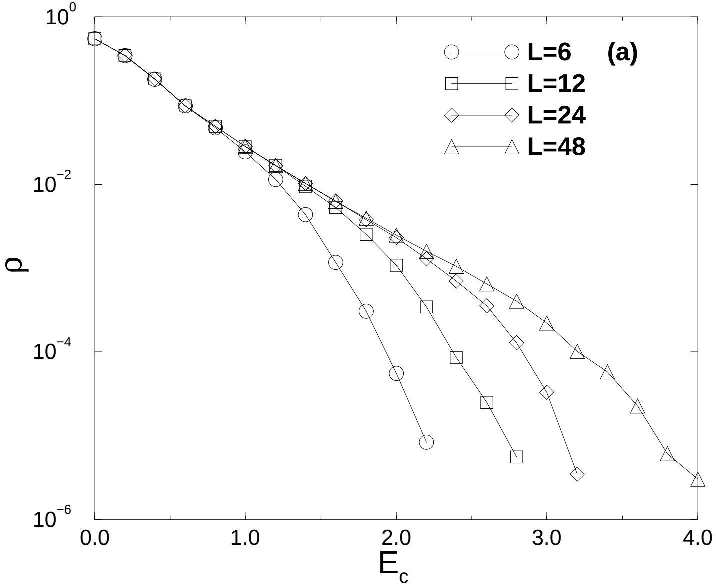
completely optimized $\sim$ **E_{pin}**

variance

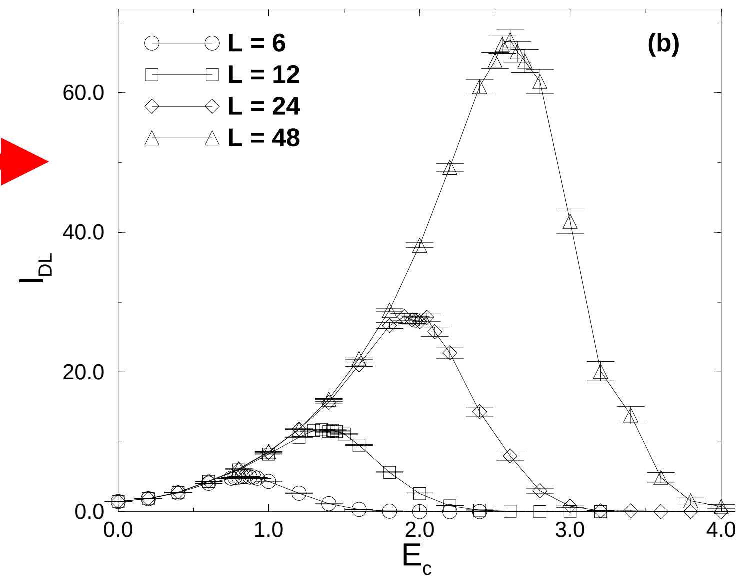
$$\sigma(\Delta E) \sim \begin{cases} \ln(L) & \text{fixed defect pair} \\ \ln^{2/3}(L) & \text{partially optimized} \\ \ln^{1/2}(L) & \text{completely optimized} \end{cases}$$

Multi-defect pairs (N>1)

vortex core energy E_C



$E_c^{\max}(L)$



$$\rho(E_c) \sim e^{-(E_c/E_0)^{\alpha}}$$

$E_c \in$	E_0	α
$[0, \infty[$	0.6 ± 0.15	0.75 ± 0.2
$[0, E_c^{\max}(L)[$	0.45 ± 0.03	1

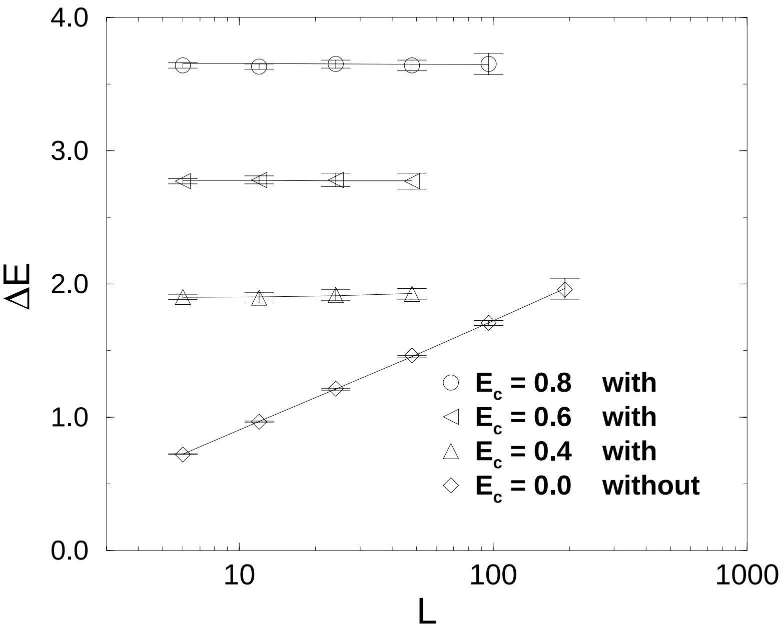
$$l_{DL}(E_c^{\max}) \sim L^{d_f}$$

$$d_f = 1.27 \pm 0.07$$

different from that of σ_c

Extra defect pair

ground state
saturated with **N** pairs



perturbation by fixed extra pair

$$\Delta E_{fix} = E_{N+1} + 2\, E_c - E_N$$

=> $\Delta E_{fix} \sim E_c$ **screening**

Summary

Summary
phase transition

Our study on **Solid-on-Solid model** exhibits ...

1. disorder-driven **flat-to-superroughtransition**
2. remarkable large **correlation length exponent** $\nu \approx$ **3.3**
3. same **universality class** as from geometrical study of contour loops
   - on random Gaussian Surfaces [Kondev et al. 1995]
   - in random elastic medium [Zeng et al.1998]

   (FPL model critical independent of disorder [Zeng et al.1998])

Summary
disloactions

Our study on **Solid-on-Solid model** exhibits ...

4. **defect energy** of fixed and optimized pair scales like in the sine-Gordon model

   [LeDoussal et al. 1998, Zeng et al. 1999]

5. **vortex core energy** exponential decay

   [Middleton 1998], ρ scales as ξ_D (< l: unpairing [LeDoussal etal.98])

6. **screening** of extra pair

   [Middleton 1998]

Outlook

- study for **unique** **height profile**
- **defect energy** and **dislocation** analysis at σ_c

  (c.f. 3D strongly screened gauge glass model)

- **finite low temperature** regime:

  combinatorial optimization + MC simulation

  (Schehr & Rieger in progress)

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