THE DYNAMICAL BEHAVIOUR OF GUSTS

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ABSTRACT:
By means of stochastic tools we describe wind gusts in a probabilistic way. In this sense gusts are defined as velocity changes over short time intervals. We show evidence that the phenomenon of gusts is strongly connected to the well known intermittent statistics of fully developed turbulence, i.e. anomalous high probability of finding high velocity fluctuations. As a further characteristic we investigate the waiting times statistics of successive wind gusts. Most interestingly we find power law statistics indicating scaling or self similar properties. On the basis of these statistics a forecast becomes difficult or even impossible.
Furthermore we present in this contribution an approach how to extract stochastic models from measured time series. As an application stochastic modeling of power output of wind turbines will be presented.

1 INTRODUCTION

Turbulence means a severe problem in wind energy industry. One prominent phenomenon related to atmospheric turbulence are the frequent occurrences of wind gusts.

In this papper we will show how gusts can be described by use of increment statistics and how they are related to laboratory turbulence. Therefore we analyze a *275 h* excerpt of a data set of atmospheric velocity recorded near the German North Sea coastline in Emden (*30 m* height, *4 Hz* sample frequency). Additionally we compare these data with a turbulent velocity field far behind a cylinder of a wind tunnel experiment where the turbulence can be considered to be stationary and nearly isotropic [1] (different to atmospheric velocity fields).

In a second part we introduce the principle relevance and possible applications of a stochastic approach (using a stochastic differential equation). This ansatz seems to be promissing to understand and to describe dynamical effects caused by the strongly turbulent velocities more basically.

2 INCREMENT STATISTICS

For atmospheric velocity fields it is common to distinguish between the mean velocity $\overline{U}$ (normally a running *10 min* average) and the turbulent fluctuations u around it [2]. In this way the component of the horizontal velocity in mean wind direction U at time t can be written as:

$$U(t) = \overline{U}(t) + u(t). \qquad (1)$$

Starting from the so defined fluctuations *velocity increments* over a certain time-distance τ can be defined as follows.

$$\begin{aligned} \delta u_\tau &:= U(t+\tau) - U(t) \\ &\approx u(t+\tau) - u(t) \end{aligned} \qquad (2)$$

The increments directly measure the fluctuation differences over the time step τ. Every increment

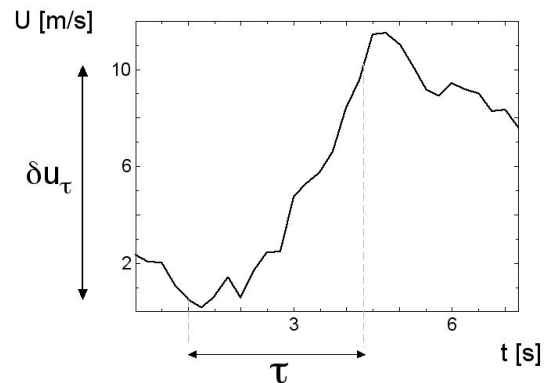exceeding a certain level - $\delta u_\tau > A$ - can be defined as a wind gust as illustrated in Fig. 1.

Fig.1: The picture shows an arbitrary excerpt of the fluctuation time series including an extrem wind gust. During 4 s the velocity increases about 10 m/s.

2.1 Probability density functions

To get information about the frequency of occurances of gusts the **p**robability **d**ensity **f**unctions (pdfs) $P(\delta u_\tau)$ of the increments are calculated.

As it can be seen in Fig. 2 these pdfs show clear non-Gaussian shapes with marked peaks around the mean value ($\delta u_\tau = 0$) and fat tails for large increments. Note that the large increments – located in the tails – correspond to strong wind gusts. As illustrated in Fig. 2 (arrow) the probability density to find these large increments for an intermittent distribution is several orders of magnitude higher than for a Gaussian one [3,4]. Interestingly the distributions stay clearly intermittent even for large τ-values, different to stationary and isotropic laboratory turbulence. In the latter a change of the shape is observed as a function of τ (see [5]). For increasing τ the distributions approach more and more to

a Gaussian one. The missing of this characteristic change of shape for the atmospheric pdfs can be considered to be the reason for the large amount of wind gusts.
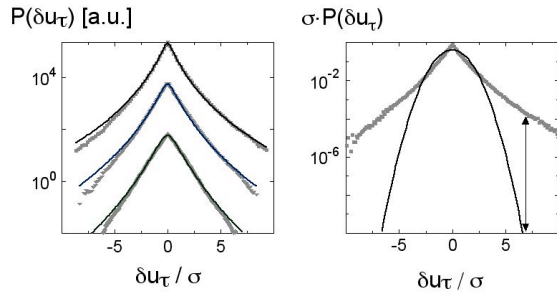

Fig. 2: In the left picture the pdfs for τ =0.25 s, 7.0 s and 35 min are shown in a semilogarithmic presentation. The filled symbols represent the measured distributions, the solid lines a fit introduced by Castaing [5]. The pdfs are shifted vertically against each other for a clearer presentation.
In the right graphic the pdf for τ =4 s is compared to a Gaussian distribution (solid line). Both distributions have the same standard deviation σ.

To explore how atmospheric and laboratory turbulence are related to each other we consider the conditioned pdfs for a fixed mean velocity $P(\delta u_{\tau} \mid \overline{U})$. This means that only those increments are taken into account for those the mean velocity $\overline{U}(t)$ (compare eq. (1)) ranges in a fixed velocity interval. In Fig. 3 the pdfs for the laboratory data as well as for the conditioned wind data are shown. In both cases a similar development of the stochastic distributions from Gaussian shapes for large τ to intermittent ones for small τ is observed.
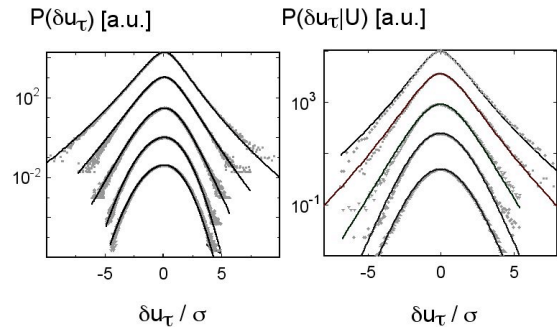

Fig. 3: In the left picture the pdfs of the wind tunnel measurement are shown for τ equal to 0.005T, 0.02T, 0.17T, 0.67T and 1.35T (from the top to the bottom) . The right side shows the distributions of the conditioned wind increments ($\overline{U}$ ranges in [4.5 ;5.6] m/s) τ being 0.008T, 0.03T, 0.2T, 0.95T and 1.9T.
In both pictures the symbols represent the measured pdfs whereas the solid lines correspond to the fit introduced by Castaing [5]. The distributions are shifted against each other.

From this finding we propose that the atmospheric turbulence seems to consist of stationary, local isotropic "turbulence packages" as observed in laboratory turbulence and wind gusts (large increments) seem to be the result of the mixing of different turbulent packages due to the instationarity of atmospheric winds.
To compare the different time scales of both data sets the time is expressed in units of T which is the integral time

defined by means of the autocorrelation function of the fluctuations:

$$T = \int_0^\infty R(\tau)d\tau \cdot \qquad (4)$$

For the laboratory data the integral time is about *6 ms*, for the wind data about *30 s*.

2.2 Waiting Times Statistics

So far we have discussed the frequency of occurrences of gusts but nothing is known about their temporal distribution. For this purpose we also consider the waiting times distribution $P(\Delta t)$ (distribution of the time-distances Δt between successive gusts).
As it is shown in the plot below we find a clear power law distribution (fractal distribution) up to time-distances of about one hour. Greater distances are not resolved due to the limited data of 275 hours. Obviously the exponent depends on the combination of the chosen threshold A and τ.
Note that for a power law distribution the moments diverge in general so that a mean value of the time-distance between successive gusts does not exist which prevents a proper estimation of Δt.
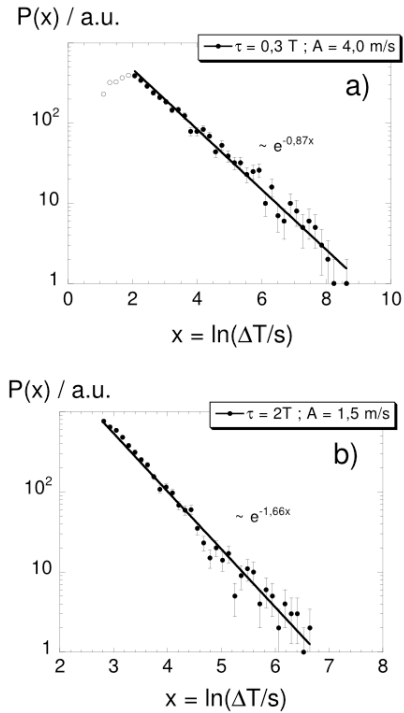

Fig. 4: The waiting times between successive gusts are plotted in a double logarithmic presentation due to $x = \ln(\Delta t)$, so that the exponential fits correspond to a power law.

3 DYNAMICAL FEATURES

So far we have explored the frequency- as well as the magnitude distribution of wind gusts. For many applications (e.g. the Annual Energy Production (AEP) of a Wind Energy Converter (WEC), loads acting the converter, etc.) it is important not only to know how often

a certain velocity interval ΔU arises but also to explore the dynamical behaviour of the velocity field.

Due to inertia effects many short ΔU-intervals might lead to a different energy output of the WEC than one long period although in both cases $P(\Delta U)$ are the same. This is illustrated in an exemplary way in Fig. 5 where the total time the velocity ranges in ΔU is the same but in the left picture U changes it's value very rapidly.
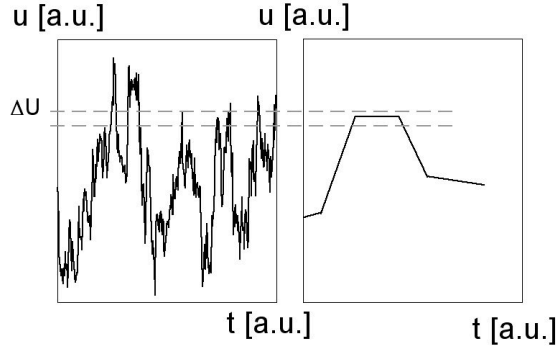

Fig. 5: The left picture shows a turbulent excerpt of the velocity time series while the rigth one is an idealized, schematical realisation of a more constant velocity.

To describe these dynamical effects we propose to consider the velocity as a stochastic variable with a time evolution described by a **Langevin-equation** [6]. For a general stochastic variable $\vec{X}(t)$ with realisations $\vec{x}(t)$ this equation reads:

$$\frac{d}{dt}X_i(t) = D_i^{(1)}(\vec{x}) + \left(\sqrt{D^{(2)}(\vec{x})}\right)_{ij} \cdot \Gamma(t) \quad . \quad (3)$$

While the first term on the right hand side of eq. (3) describes the deterministc evolution of $\vec{X}(t)$ the second reflects the influence of noise ($\Gamma(t)$ stands for δ-correlated white noise). $D^{(1)}$ is called *drift-* and $D^{(2)}$ *diffusion coefficient* and they can be evaluated by the conditional moments [6].

$$D_i^{(1)} = \lim_{\tau \to 0}\frac{1}{\tau}\left\langle X_i(t+\tau) - x_i \right\rangle\Big|_{\vec{X}(t)=\vec{x}}$$

$$(4)$$

$$D_{ij}^{(2)} = \lim_{\tau \to 0}\frac{1}{\tau}\left\langle (X_i(t+\tau) - x_i)(X_j(t+\tau) - x_j) \right\rangle\Big|_{\vec{X}(t)=\vec{x}}$$

In this way $D^{(1)}$ and $D^{(2)}$ can be calculated directly from the data. If $\vec{X}$ is a two-dimensional vector the deterministic as well as the stochastic (noisy) relations between both components can be obtained quite easily.

As an example consider the velocity $U(t)$ and the power $L(t)$ of a WEC to be the two components of the stochastic variable. In Fig. 6 the drift coefficient $\vec{D}^{(1)}(U,L)$ for two different WEC is shown. In Fig. 6 a) data from a 200 kW WEC (Vestas V25, WME Project, Fehmarn, Germany) with a resolution of one hour and in Fig.6 b) data from a small 250 W converter with a resolution of *20 Hz* are used [7].

As a new quality we obtain here the dynamical responses of the WEC to wind fluctuations, which are visualized by the arrows. The size of the arrows indicates the speed with which the WEC reacts on a wind fluctuation.

As another example the velocity field itself may be investigated. Considering the horizontal fluctuations parallel (u_1) and perpendicular (u_2) to the mean wind we find a very small drift around the state (0,0). While for most points (0,0) is attractive, i.e. fluctuations have the tendency to decay, we see for small and positive u_1 and large u_2, that u_1 tends to increase (see grey region in Fig. 7). This reveals a dynamical asymmetry of the horizontal fluctuations that would have been unrecognized by only plotting u_2 vs. u_1.
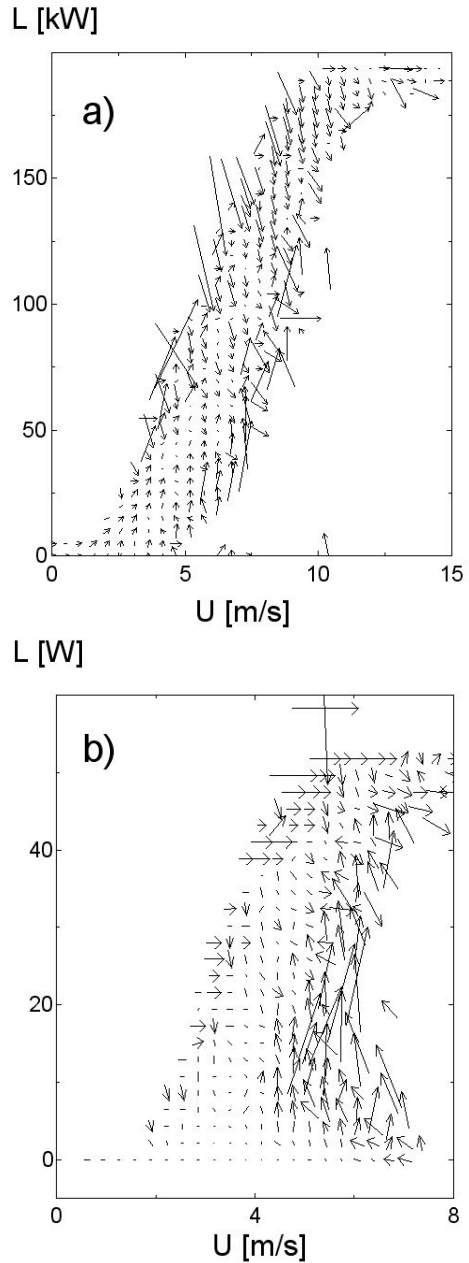

Fig.6: Vectorplot of $\vec{D}^{(1)}(U,L)$ for a low and a high resolved data set are illustrated in a) and b) respectively.
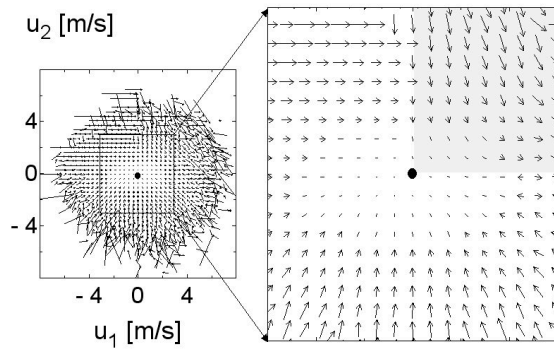
Fig. 7: The driftcoefficient of the two horizontal fluctuation components is illustrated. The right picture represents a zoom into the area around the point (0,0).

4 CONCLUSIONS - OUTLOOK

In the first part we have shown how increment statistics can be used to describe wind gusts in an adequate way concerning their magnitude as well as their temporal distribution. Furthermore the relation between the stationary and isotropic turbulence of wind tunnel experiments to atmospheric turbulence has been discussed. The latter seems to be consistent with the laboratory one when a proper condition on the mean wind speed is used.

In the second part we have introduced the relevance of a stochastic approach using the Langevin-equation. In this way the velocity and velocity-dependant quantities such as for instance the power output of a WEC can be devided into a deterministic and a stochastic (noisy) part. Doing so the dynamical reactions of a WEC on turbulent fluctuations (gusts) of atmospheric winds can be taken into account more properly than by only considering averaged quantities as it is usually done.

Thus we have presented a new tool to investigate the dynamical behaviour of instruments used for wind energy applications.

5 REFERENCES

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