

Black Holes in Alternative Theories of Gravity

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Outline

- 1 Introduction
- 2 EsGB BHs
 - EdGB BHs
 - EsGB BHs
- 3 EsGB+R BHs
- 4 Conclusions



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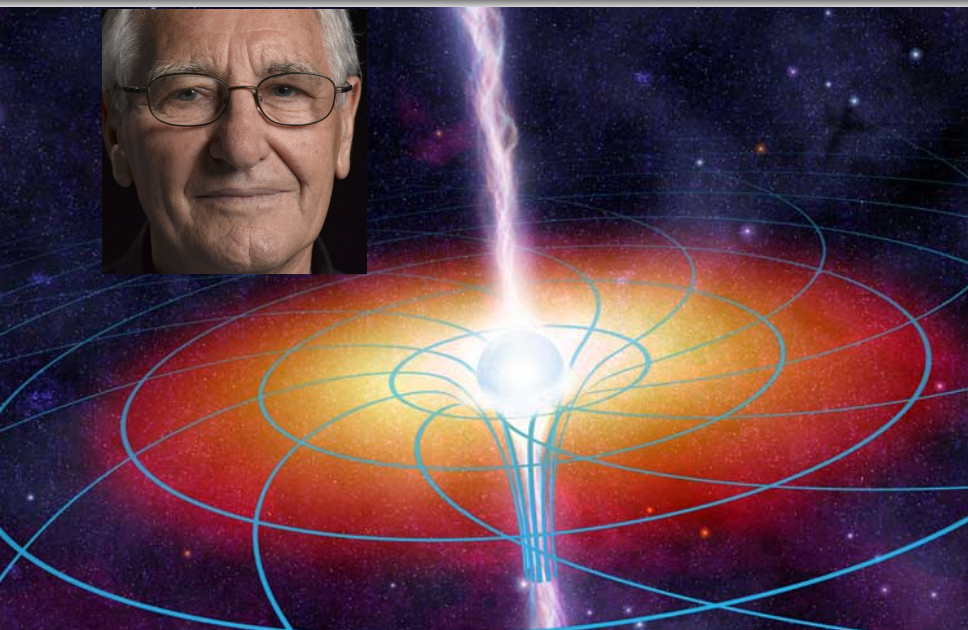


Outline

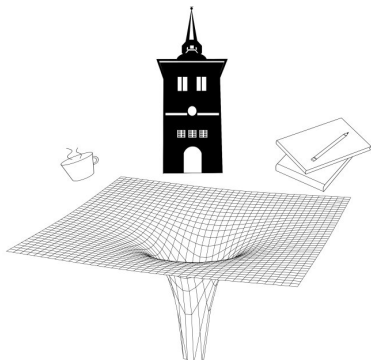
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GR: Kerr black holes



GR: Kerr black holes

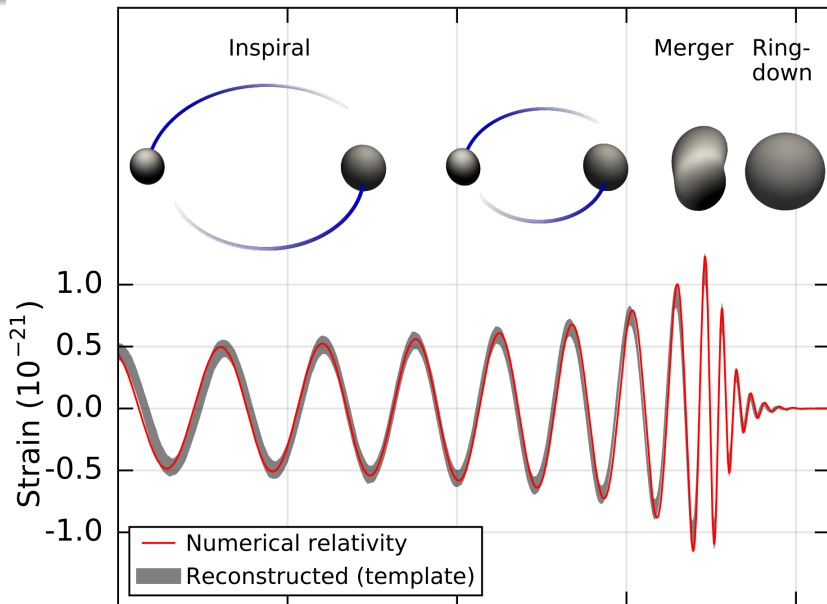


A Kerr black hole has no hair

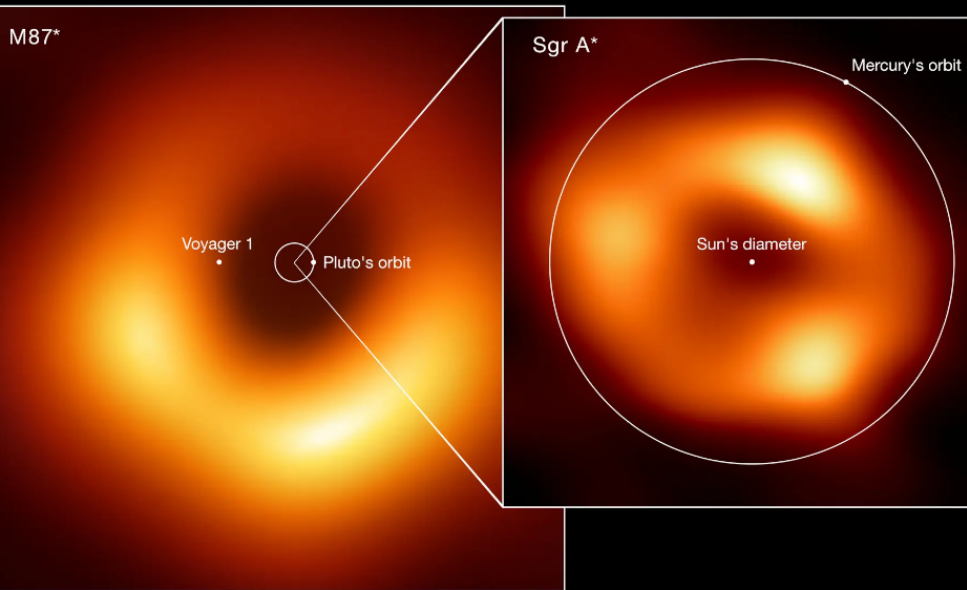
A Kerr black hole is fully characterized in terms of only two global parameters: the mass M and the angular momentum J

$$\frac{J}{M^2} \leq 1$$

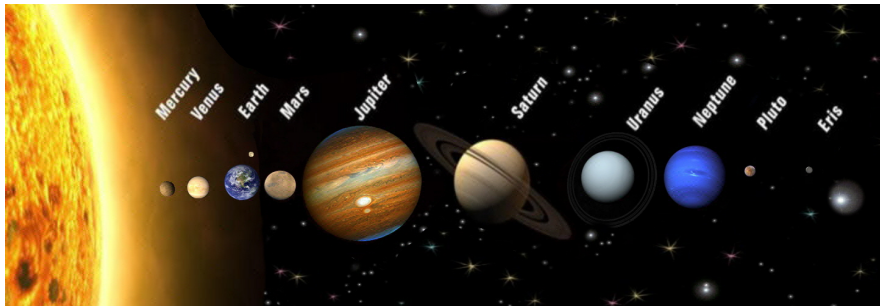
GR: Kerr black holes



GR: Kerr black holes



Alternative Theories of Gravity



- Compatible with all solar system tests!
- Strong gravity?
 - Black holes
 - Neutron stars
 - Exotic compact objects
- Cosmology?



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Einstein-scalar-Gauss-Bonnet Theories

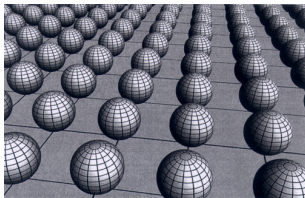
EsGB action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \varphi)^2 + f(\varphi) R_{\text{GB}}^2 \right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R_{\text{GB}}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$$

coupling function $f(\varphi)$



The resulting set of equations of motion are of second order (Horndeski).

Einstein-scalar-Gauss-Bonnet Theories



Gregory Horndeski, 'Horndeski Scalar Theory, Past, Present and Future'

Einstein-scalar-Gauss-Bonnet Theories

generalized Einstein equations

$$\begin{aligned}
 G_{\mu\nu} &= -\frac{1}{4}g_{\mu\nu}\partial_\rho\varphi\partial^\rho\varphi + \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi \\
 &- \frac{1}{2}(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu})\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}_{\alpha\beta}\nabla_\gamma\partial_\kappa f(\varphi)
 \end{aligned}$$

scalar equation

$$\nabla_\mu\nabla^\mu\varphi + \frac{df}{d\varphi}R_{\text{GB}}^2 = 0$$

crucial: choice of coupling function $f(\varphi)$

- GR black hole solutions do not remain solutions
 \implies only hairy black holes result
- GR black hole solutions do remain solutions
 \implies in addition scalarized black holes emerge

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EdGB black holes

Kanti et al. hep-th/9511071, Torii et al. gr-qc/9606034

coupling function

$$f(\phi) = \frac{\alpha}{4} e^{-\gamma\phi}$$

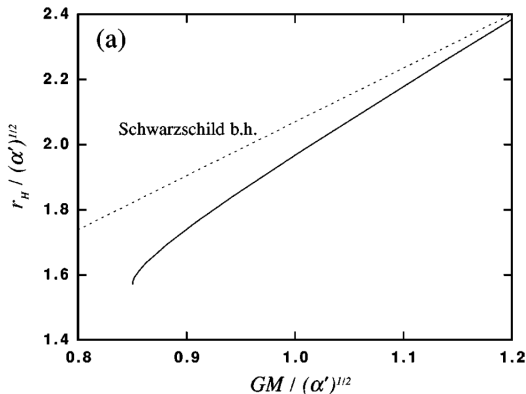
static black holes

critical black holes:

horizon expansion

$$\sqrt{1 - 6 \frac{\alpha'^2}{r_h^4} e^{2\phi_h}}$$

lower bound
on the horizon size
for fixed α'

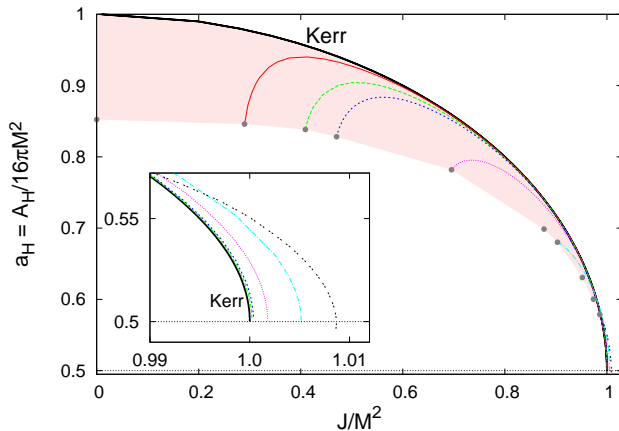


lower bound on the mass

EdGB black holes

Kleihaus et al. 1101.2868

horizon area versus angular momentum



EdGB black holes

Cunha et al. arXiv:1701.00079

shadow



$$\alpha/M^2 = 0.172, J/M^2 = 0.41$$



EdGB black holes

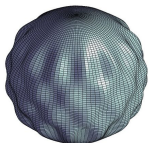
perturbation theory: damped oscillations

metric

$$g_{\mu\nu} = g_{\mu\nu}^{(0)}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \varphi)$$

scalar

$$\phi = \phi_0(r) + \epsilon \delta\phi(t, r, \theta, \varphi)$$



polar modes: even-parity perturbations

axial modes: odd-parity perturbations (pure space-time modes)

master equation: Schrödinger-like equation

eigenvalue ω

$$\omega = \omega_R + i\omega_I$$

frequency: ω_R

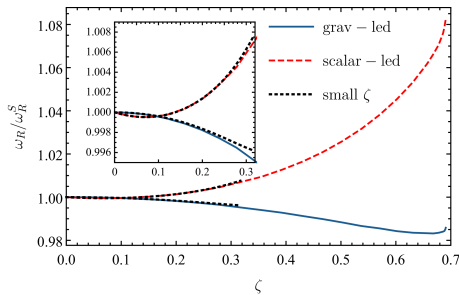
decay time: $\tau = 1/\omega_I$

EdGB black holes

Blazquez-Salcedo et al. 1609.01286

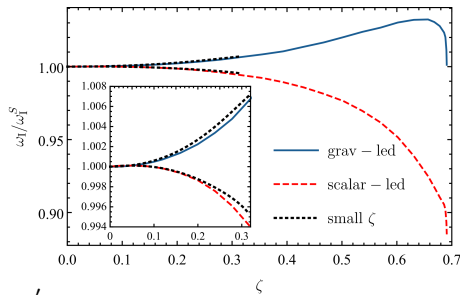
quasi-normal mode (polar $l = 2$) versus coupling constant

normalized to the Schwarzschild values



real part

$$\zeta = \frac{\alpha'}{M^2}$$



imaginary part

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Static curvature induced scalarized black holes

Doneva et al. 1711.01187, Silva et al. 1711.02080, Antoniou et al. 1711.03390

curvature induced scalarized black holes

Einstein equations

$$G_{\mu\nu} = T_{\mu\nu}$$

scalar equation

$$\nabla_{\mu} \nabla^{\mu} \varphi + \frac{df}{d\varphi} R_{\text{GB}}^2 = 0$$

GR solutions remain solutions: $\varphi = 0$, $\frac{df(\varphi)}{d\varphi} = 0$

Gauss-Bonnet: Schwarzschild

$$R_{\text{GB}}^2 = \frac{48M^2}{r^6} > 0$$

tachyonic instability

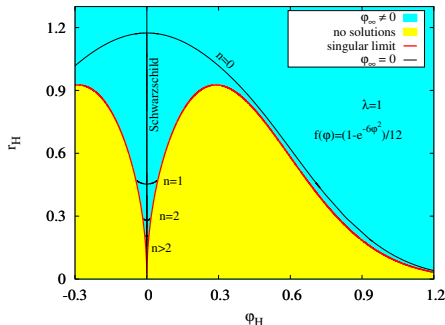
effective mass

$$m_{\text{eff}}^2 = -\eta R_{\text{GB}}^2 < 0, \quad \text{if } \eta > 0$$

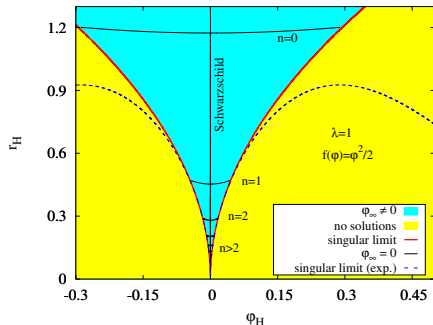
Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755

domain of existence of spontaneously scalarized static black holes



$$f(\varphi) = \frac{\lambda^2}{12} \left(1 - e^{-6\varphi^2} \right)$$

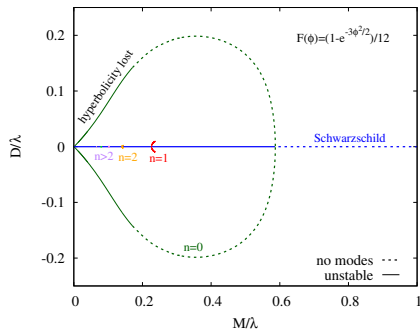


$$f(\varphi) = \frac{\lambda^2}{2} \varphi^2$$

spontaneously scalarized black holes, $\varphi_\infty \neq 0$, radicand negative

Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755



solutions

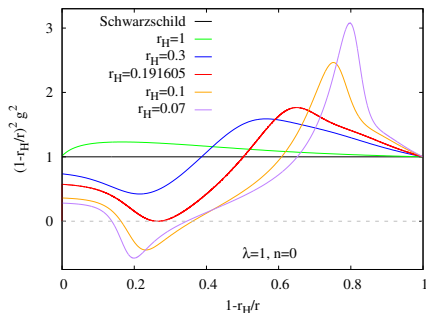
Schwarzschild blue

scalarized $n = 0$ dark green

scalarized $n > 0$...

scalar equation

$$g^2(r)\ddot{\varphi}_1 - \varphi_1'' + C_1(r)\varphi_1' + U(r)\varphi_1 = 0$$

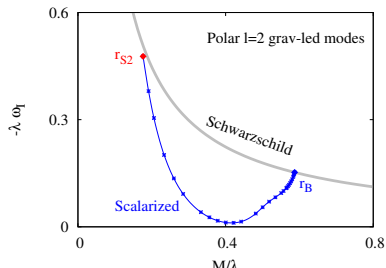
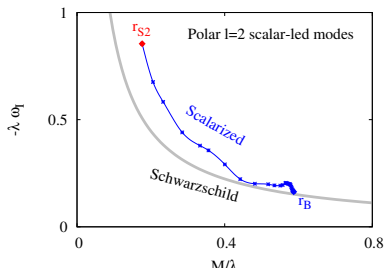
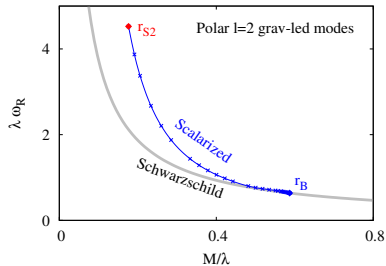
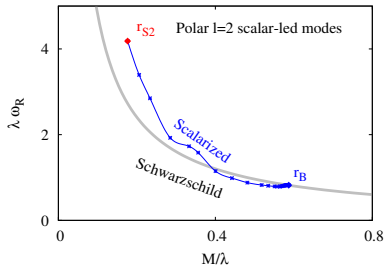


$$\left(1 - \frac{r_H}{r}\right)^2 g^2 \text{ vs } 1 - r_H/r$$

lost hyperbolicity

Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 2006.06006



Rotating curvature induced scalarized black holes

Cunha et al. 1904.09997, Collodel et al. 1912.05382, Dima et al. 2006.03095

scalar equation

$$\nabla_{\mu} \nabla^{\mu} \varphi + \frac{df}{d\varphi} R_{\text{GB}}^2 = 0$$

Gauss-Bonnet: Kerr

$$R_{\text{GB}}^2 = \frac{48M^2}{(r^2 + \chi^2)^6} (r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6) , \quad \chi = a \cos \theta$$

effective mass

$$m_{\text{eff}}^2(r) = -\eta R_{\text{GB}}^2 < 0$$

- $\eta > 0$

\implies spin suppresses scalarization

- $\eta < 0$

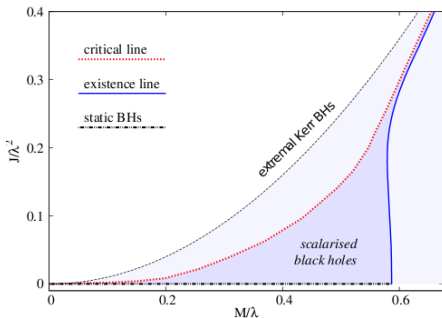
\implies spin induces scalarization

Rotating curvature induced scalarized black holes

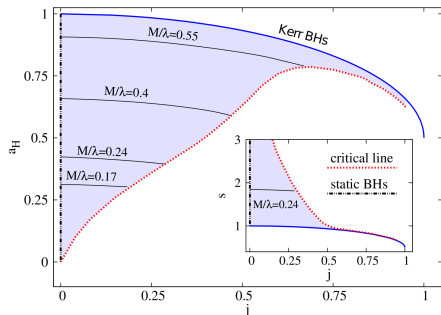
Cunha et al. arXiv:1904.09997

coupling function

$$f(\varphi) = \frac{\lambda^2}{12} \left(1 - e^{-6\varphi^2}\right), \quad \eta > 0, \quad V(\varphi) = 0$$



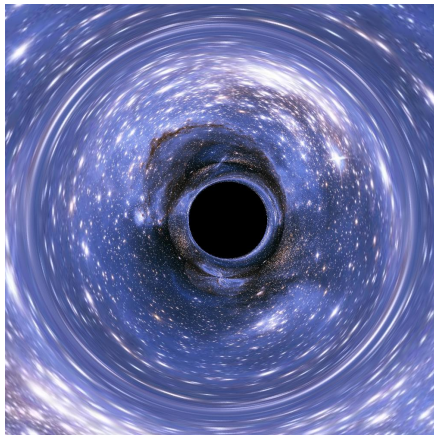
angular momentum vs mass



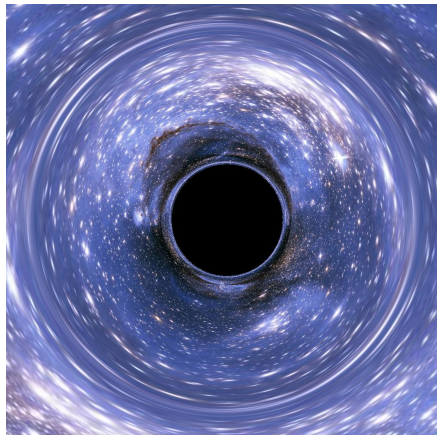
area/entropy vs angular momentum

Rotating curvature induced scalarized black holes

Cunha et al. arXiv:1904.09997



EsGB



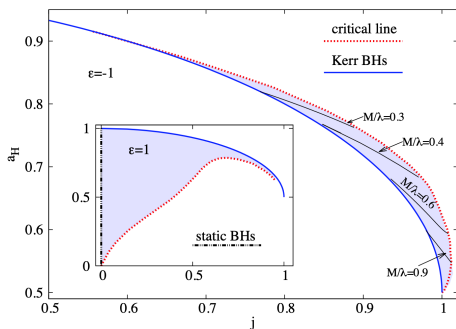
Kerr

$$M/\lambda = 0.237(j = 0.24)$$

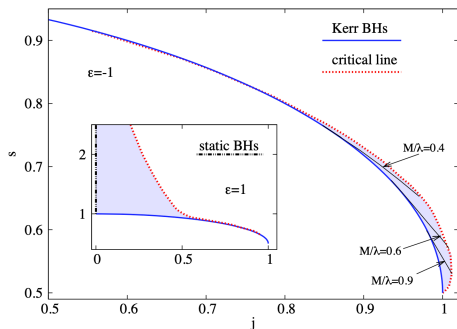
Rotating spin induced scalarized black holes

Herdeiro et al. [arXiv:2009.03904](https://arxiv.org/abs/2009.03904), Berti et al. [arXiv:2009.03905](https://arxiv.org/abs/2009.03905)
coupling function

$$f(\varphi) = \frac{\lambda^2}{12} \left(1 - e^{-6\varphi^2} \right), \quad \eta < 0, \quad V(\varphi) = 0$$



area vs angular momentum

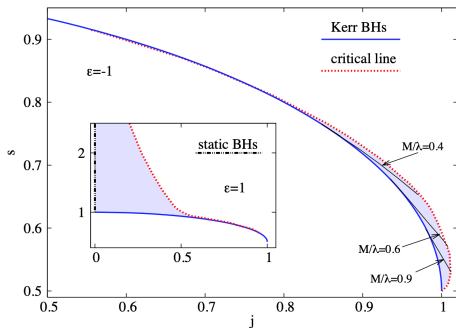


entropy vs angular momentum

even scalar field

Rotating spin induced scalarized black holes

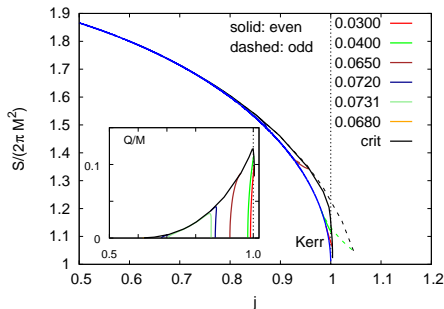
Herdeiro et al. arXiv:2009.03904



entropy vs angular momentum

$$f(\varphi) = \frac{\lambda^2}{12} \left(1 - e^{-6\varphi^2}\right)$$

Berti et al. arXiv:2009.03905



entropy vs angular momentum

$$f(\varphi) = \frac{\lambda^2}{8} \varphi^2$$

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Einstein-scalar-Gauss-Bonnet with Ricci coupling

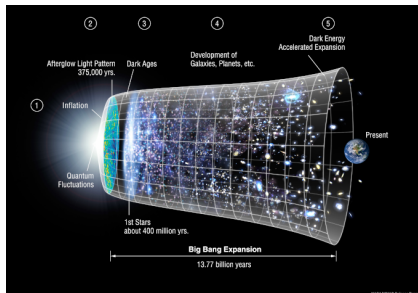
Antoniou et al. 2004.14985

Compact object scalarization with general relativity as a cosmic attractor

EsGB with Ricci action

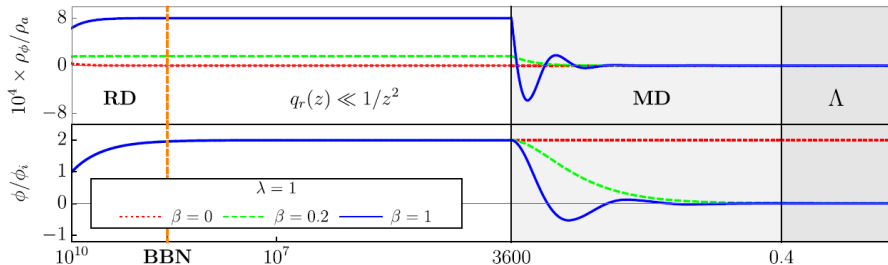
$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial_\mu \varphi)^2 + \frac{\varphi^2}{2} \left(\alpha R_{\text{GB}}^2 - \frac{\beta}{2} R \right) \right]$$

coupling function $f(\varphi) = \frac{\varphi^2}{2}$



Einstein-scalar-Gauss-Bonnet with Ricci coupling

Antoniou et al. 2004.14985



top: energy density ratio of scalar ρ_ϕ and cosmic fluid ρ_a vs redshift z

bottom: evolution of scalar field ϕ in units of its initial value ϕ_i

RD: radiation dominated, MD: matter dominated, Λ : Λ dominated

$$m_{\text{eff}}^2 = \frac{\beta}{2}R - \alpha R_{\text{GB}}^2$$

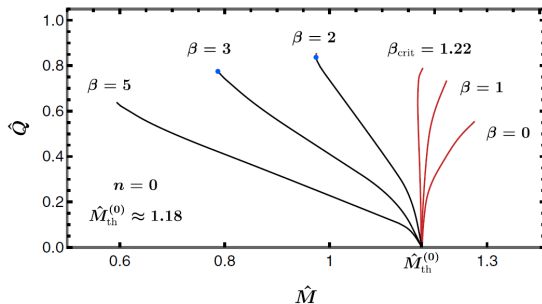
Static curvature induced scalarized black holes

Antoniou et al. 2105.04479

domain of existence of spontaneously scalarized static black holes

$$m_{\text{eff}}^2 = \frac{\beta}{2}R - \alpha R_{\text{GB}}^2 < 0$$

tachyonic instability: independent of β ($R = 0$)



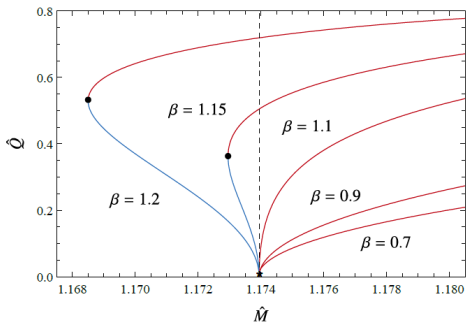
scaled scalar charge vs scaled mass for varying Ricci coupling β

endpoint: onset of instability

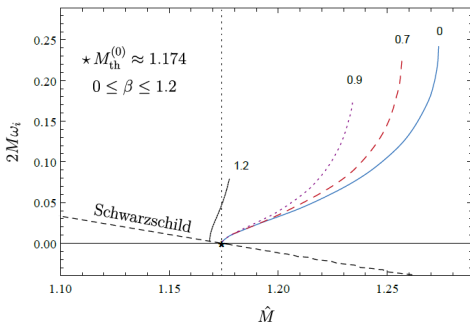
Static curvature induced scalarized black holes

Antoniou et al. 2204.01684

stability of Schwarzschild and spontaneously scalarized static black holes



charge vs mass



radial mode vs mass

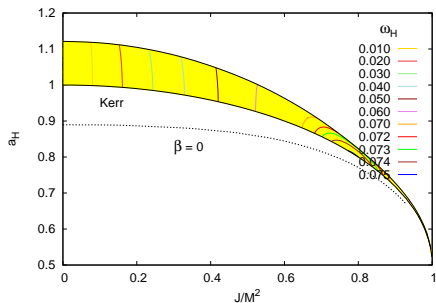
- Schwarzschild black holes unstable vor $\hat{M} < 1.174$
- scalarized black holes always unstable for $\beta = 0, 0.7, 0.9$
- scalarized black holes in part radially stable for $\beta = 1.2$

Rotating curvature induced scalarized black holes

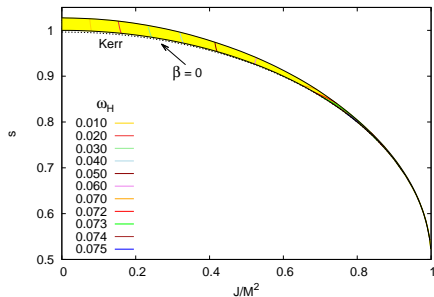
preliminary results

Ricci coupling

$\beta = 5$



angular momentum vs mass



area/entropy vs angular momentum

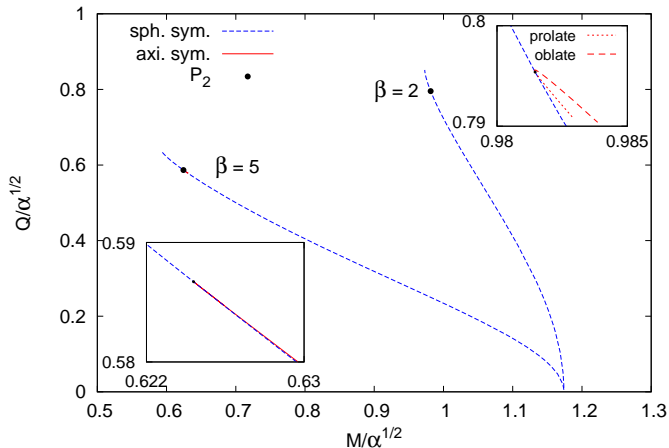
scalarized black holes are entropically preferred

Quadrupole instability of static scalarized black holes

Kleihaus et al. 2303.04107

Ricci coupling

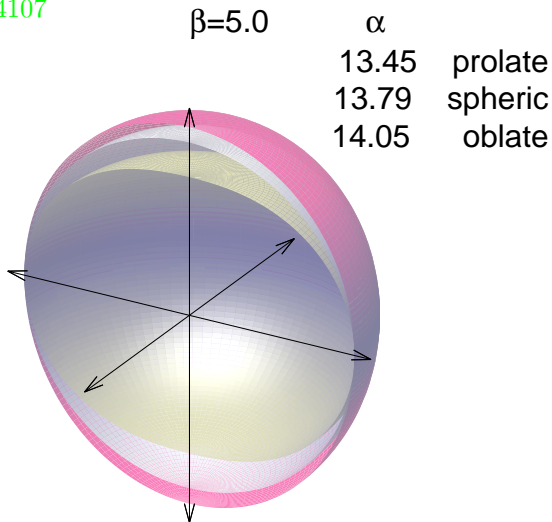
$\beta = 2$ and $\beta = 5$



scalar charge vs mass

Quadrupole instability of static scalarized black holes

Kleihaus et al. 2303.04107

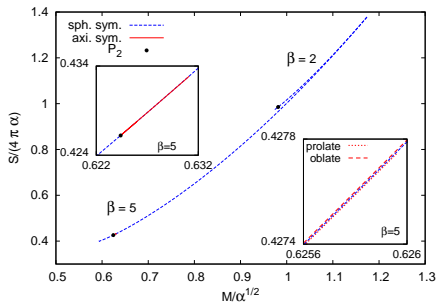


Quadrupole instability of static scalarized black holes

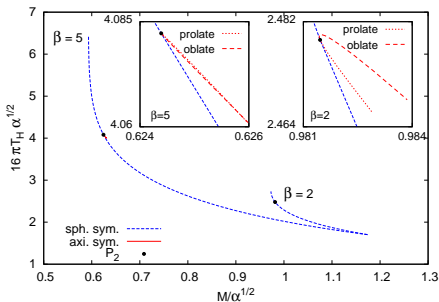
Kleihaus et al. 2303.04107

Ricci coupling

$\beta = 2$ and $\beta = 5$



entropy vs mass



temperature vs mass

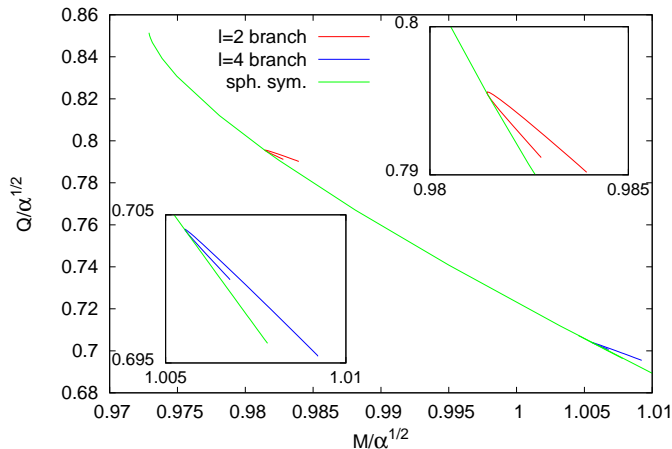
oblate scalarized black holes entropically preferred?

Hexadecupole instability of static scalarized black holes

preliminary results

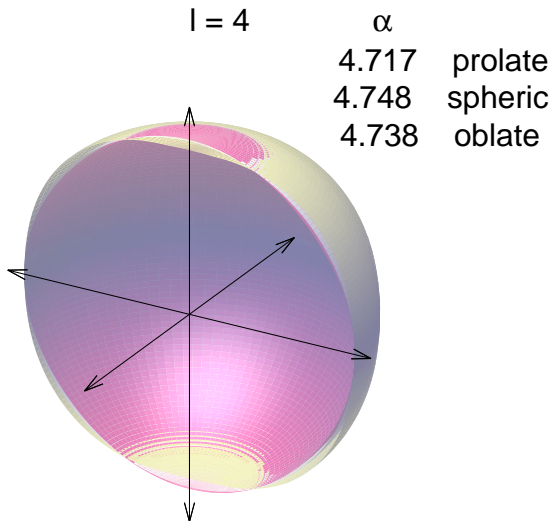
Ricci coupling

$$\beta = 2$$

scalar charge vs mass: $l = 0, l = 2, l = 4$

Hexadecupole instability of static scalarized black holes

preliminary results

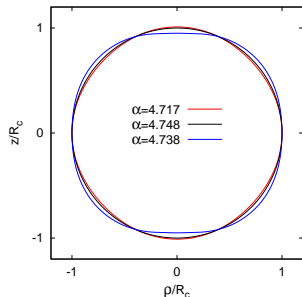


Hexadecupole instability of static scalarized black holes

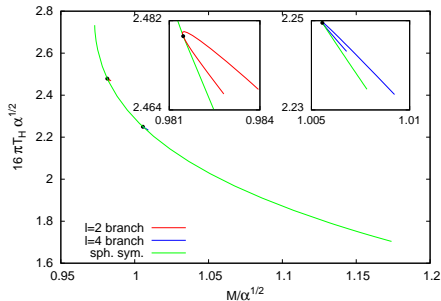
Kleihaus et al. 2303.04107

Ricci coupling

$\beta = 2$



2-dimensional embedding



temperature vs mass

next? $l = 6, l = 8, \dots ?$

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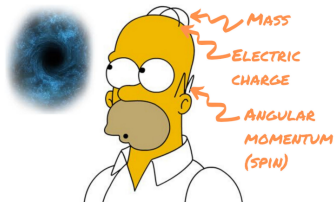


Conclusions

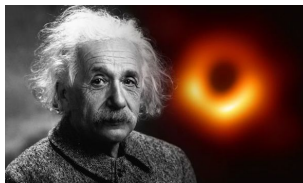
GR versus generalized gravity theories

GR black holes

- Kerr: no hair
- gravitational waves
- shadow
- ...



black holes beyond GR



- EsGB
 - dilatonic
 - spontaneously scalarized
- EsGB+Ricci
 - stability?
- ...

THANKS

