Loop length distributions in the Negative Weight Percolation problem: Extension to d=2...6

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Negative-weight percolation (NWP)

d-dimensional lattice model with quenched disorder. Edge weights ω drawn from disorder distribution

$$P(\omega) = \rho\, e^{-\omega^2/2}/\sqrt{2\pi} \;+\; (1-\rho)\;\delta(\omega-1)$$

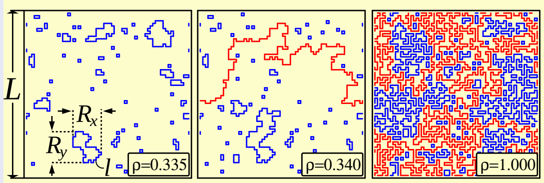
- Allows for loops $\mathcal{L}$ with negative weight $\omega_{\mathcal{L}}$
- Tunable disorder parameter ρ:
  $\rho=0$: no loops with $\omega_{\mathcal{L}} < 0$
  $\rho=1$: many (large) loops with $\omega_{\mathcal{L}} < 0$

Compute loop configuration $\mathcal{C}$ with minimum weight

$$E \equiv \sum_{\mathcal{L}\in\mathcal{C}} \omega_{\mathcal{L}} \stackrel{!}{=} \min$$

- "Easy" combinatorial optimization problem
- Solvable through mapping to minimum weight perfect matching (MWPM) problem [1]

Percolation phenomenon



(2D square lattice, side length $L=64$)

- Loop lengths l "grow" for increasing ρ
- System spanning loops appear for $\rho \geq \rho_c = 0.34$
- Disorder induced, geometric transition

Measuring and quantities

- Set up random edge weights for $\rho < \rho_c$
- Compute distribution $n(l)$ of loop lengths l
- Count "small", i.e. non-percolating loops only!

Consider the following relations:

- $\rho \approx \rho_c$: $n \propto l^{-\tau}$ with Fisher exponent τ
- $\rho < \rho_c$: $n \propto l^{-\tau}\exp(-T_L l)$ with line tension T_L
- $\rho < \rho_c$: $T_L \propto |\rho-\rho_c|^{1/\sigma}$ with the finite-size cut-off parameter σ

Estimate $\sigma_{lit} = \frac{1}{d_f \cdot \nu}$ (see [5]) employing:

- Fractal dimension d_f
- Correlation length exponent ν

Measured for percolating loops at $\rho > \rho_c$

Bibliography

[1] OM, AKH, New J. Phys. 10 (2008) 043039
[2] R. Ahuja, T. Magnanti, J. Orlin, *Network flows*, (Prentice Hall, 1993)
[3] AKH, *Practical Guide to Computer Simulations*, (World Scientific, 2009)
[4] W. Cook, A. Rohe, INFORMS 11 (1999) pp. 138-148
[5] L. Apolo, OM, AKH, Phys. Rev. E 79 (2009) 031103

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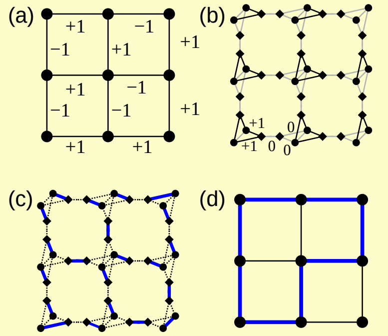
Loop algorithm – path-to-matching transformation

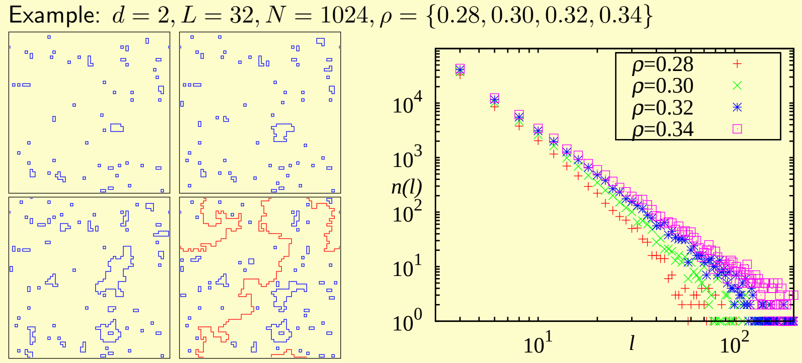


- Standard minimum-weight path algorithms don't work, since $d(i) = \min_{j\in N(i)}\left[d(j)+\omega(i,j)\right]$ not fulfilled
- Minimum-weight path problem requires matching techniques [1, 2]:
  (a) original graph G,
  (b) auxiliary graph G_A,
  (c) MWPM on G_A (blue edges),
  (d) loop configuration on G

A MWPM is found through the Blossom IV algorithm [4].

Exponential suppression of loop lengths

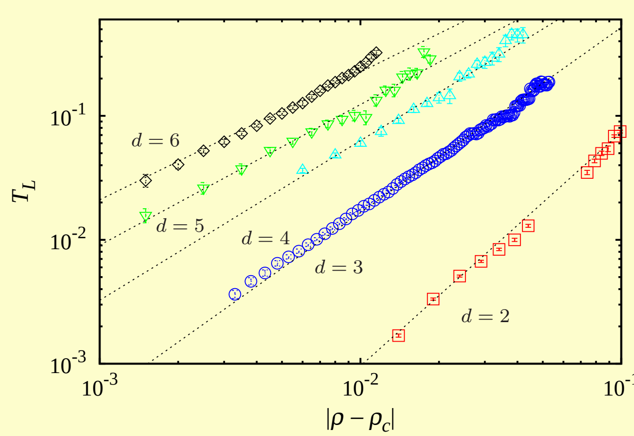
Example: $d=2, L=32, N=1024, \rho=\{0.28, 0.30, 0.32, 0.34\}$



- Distributions $n(l)$ show exponential suppression of lengths
- Measured through the $\exp(-T_L l)$ factor ($\rho=\rho_c : T_L = 0$)

Results and conclusion

d	L	N_{max}	ρ_c	σ_{lit}	best σ	Remarks about the fit
2	512	6400	0.34	0.53(3)	0.53(3)	rs for $\rho < 0.28$, $\tilde{\rho}_c = 0.344(2)$
3	64	4800	0.1273	0.69(2)	0.67(1)	$\tilde{\rho}_c = 0.1278(1)$
4	16-21	32000	0.064	0.78(3)	0.78(2)	resampling for $\rho < 0.045$
5	10-12	16000	0.0385	0.86(4)	0.88(2)	$[0.025{:}\rho_c]$
6	6	54864	0.02670	1.00(3)	0.97(4)	$[0.022{:}\rho_c]$



- Conclusion: Theory holds well up to $d=6$, fitted values meet expectations
- No considerable result at $d=7$ (upper critical dimension $d=6$, see [5]) System size limited to $L=5$ by computer memory (MWPM calculations)
- Improvement: Logarithmic binning reduces noise in the $n(l)$ tails