

بہ نام خدای که در این مرد
شکست



Discussing magnetization
of a disordered spin
lattice

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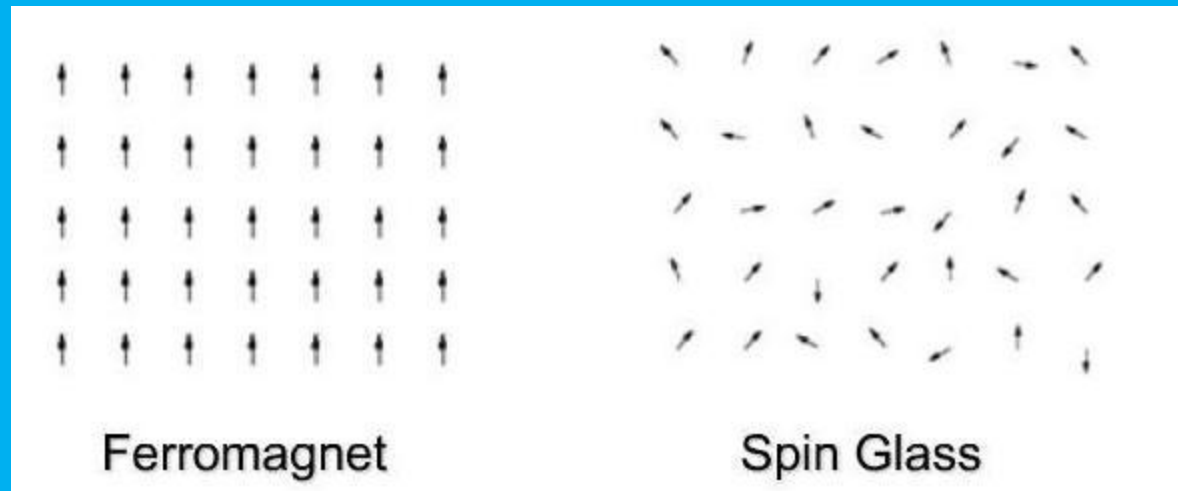
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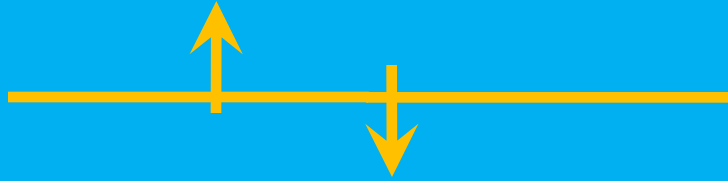
Introduction:

In this thesis ,by approach of statistical physics , we discusse the behavior of Spin glass network.

$$H = -\sum_{\langle ij \rangle} J_{i,j} \sigma_i \sigma_j + \sum_i h \sigma_i$$



As a **disorder**, we used Fractional Gaussian Noise (fGn series).

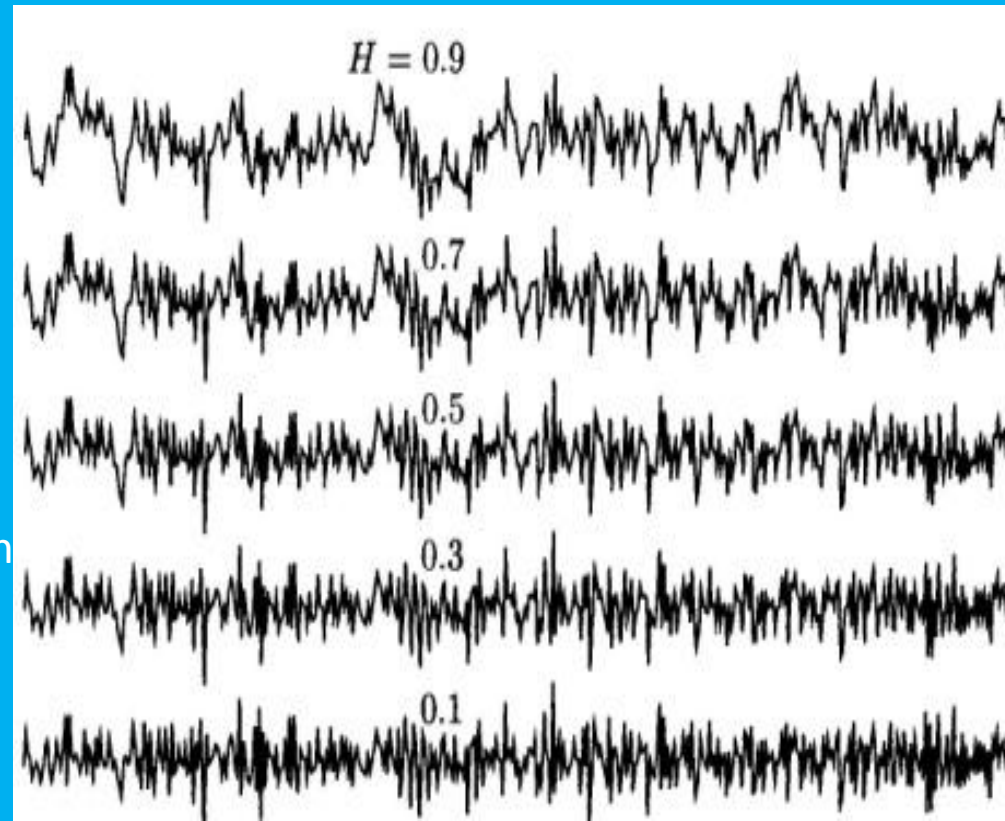


$$H = - \sum_{\langle ij \rangle} J_{i,j} \sigma_i \sigma_j$$

Persistent noise: **H>0.5** positive correlation

Random noise: **H=0.5**

Antipersistent noise **H<0.5** negative correlation



Our aim is studying the different effects of irregularities on system which we can observe these efficacies through the magnetism and susceptibility by studying the dependency between phase transition point and Hurts exponents.

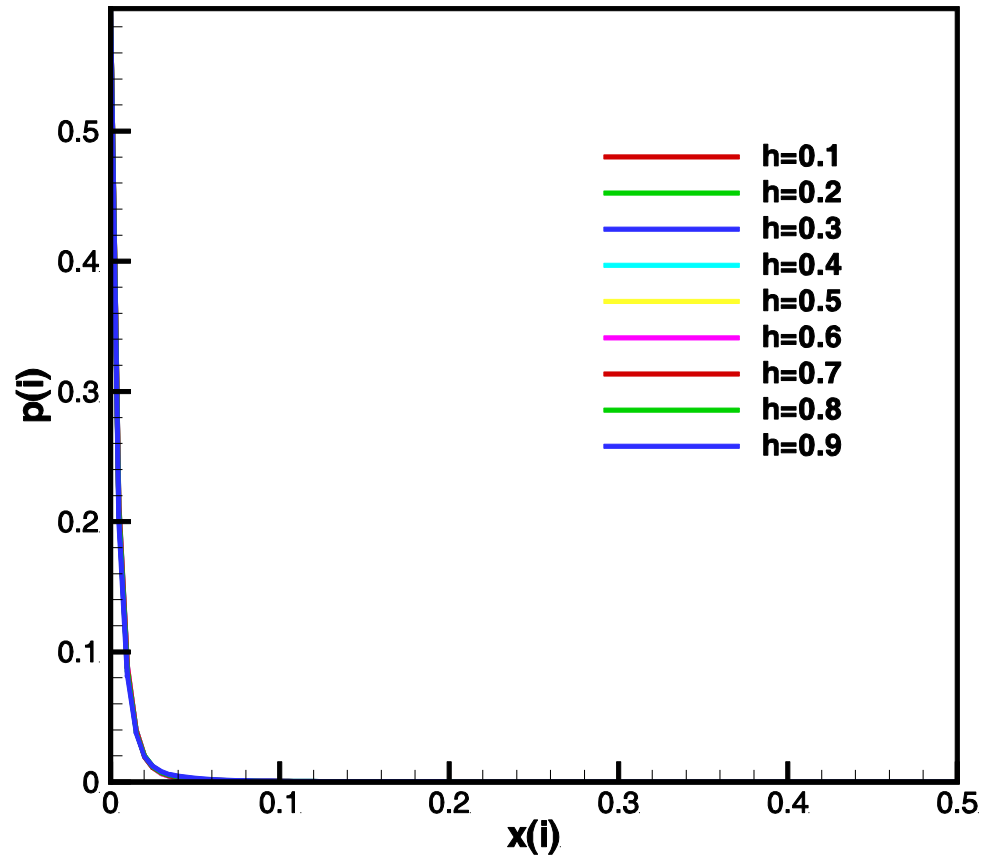
Our way of working:

First we use Random Matrix Theory (RMT) to recognize the behaving pattern of this irregularities.

Then by Monte Carlo simulation, we studied the dynamic behavior of system by those various irregularities.

This is the distribution function of level spacing diagram by using RMT for $N=1000$ which we averaged it for 1000 samples.

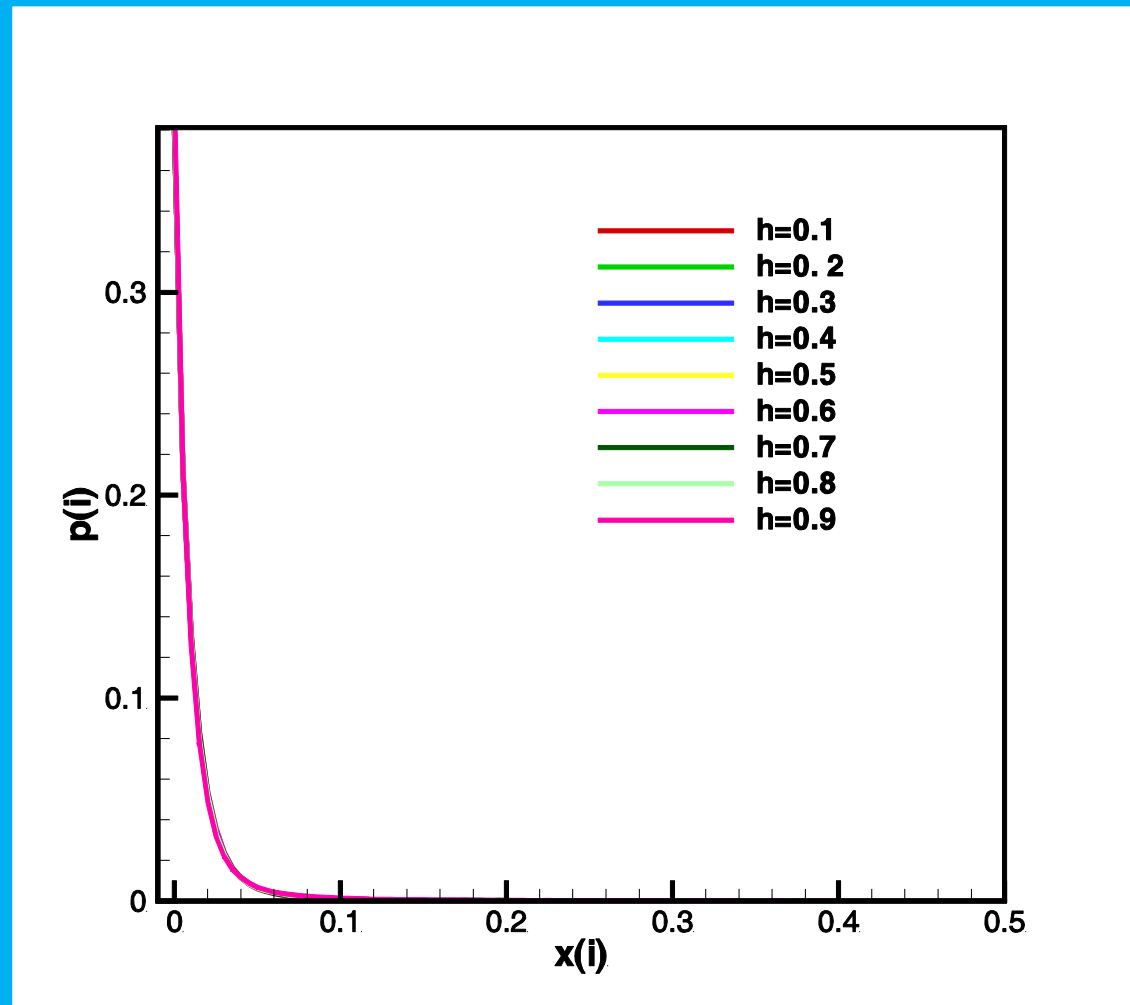
Distribution of Level spacing are the same in Different exponents.



This is the distribution function of level spacing diagram by using RMT for $N=500$ which we averaged it for 2000 samples.

Due to the same result of different sizes ($N=500, N=1000$), we conclude that distribution of level spacing does not depend on the size of net.

The same behavior of the distribution function of level spacing indicate that random matrixes of different exponent belong to the same class of random matrixes



For discussing the efficacy of various exponents on the eigen vectors ,we use IPR quantity .

As you know we can obtain the quantity IPR (Inverse Participation Ratio) from:

$$IPR = \frac{\sum |v|^4}{\{\sum |v|^2\}^2}$$

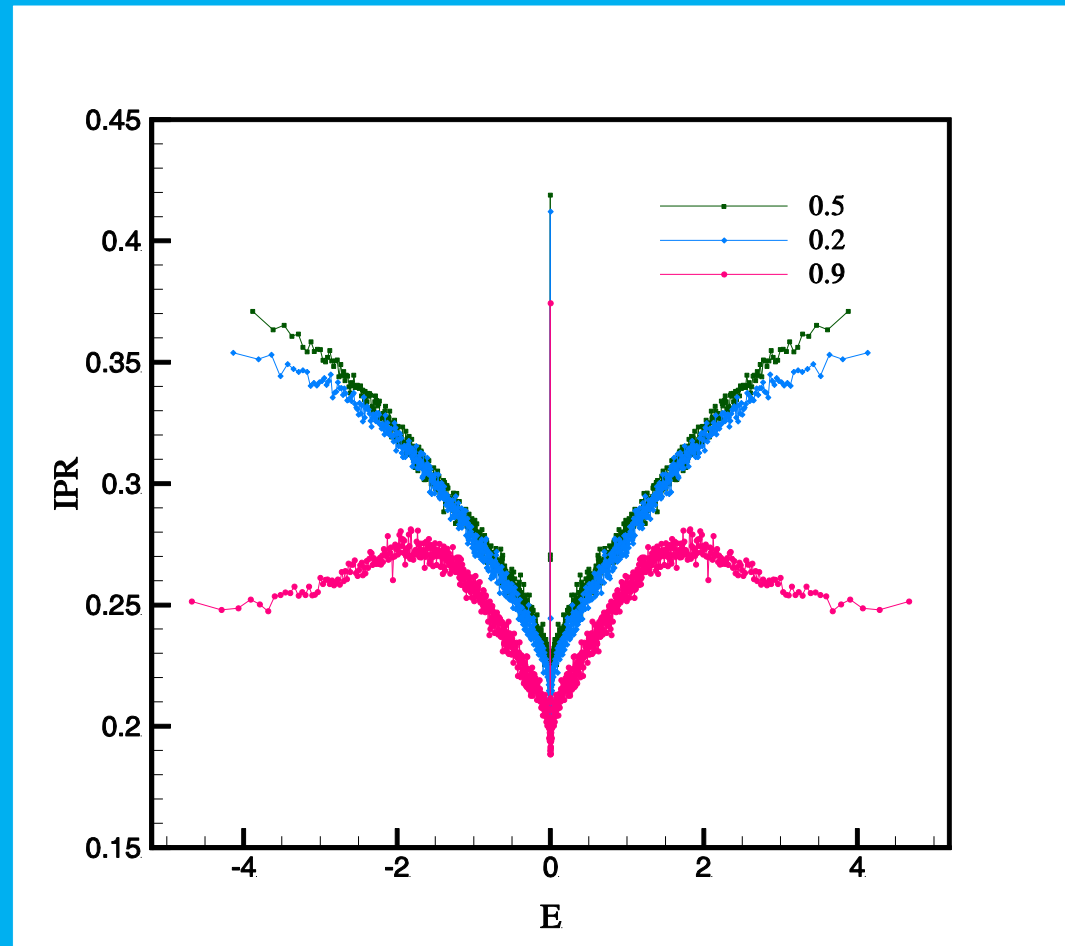
V is an eigen value.

This quantity is an indicator of location distribution of eigen vectors.

This is the IPR chart versus E (eigen value).
 $N=1000$ in 3 Hurts exponents $H=0.2, 0.5, 0.9$
which was averaged by 1000 samples.

As we can see the behavior
of the eigen vectors
changes due to the
changing of exponents.

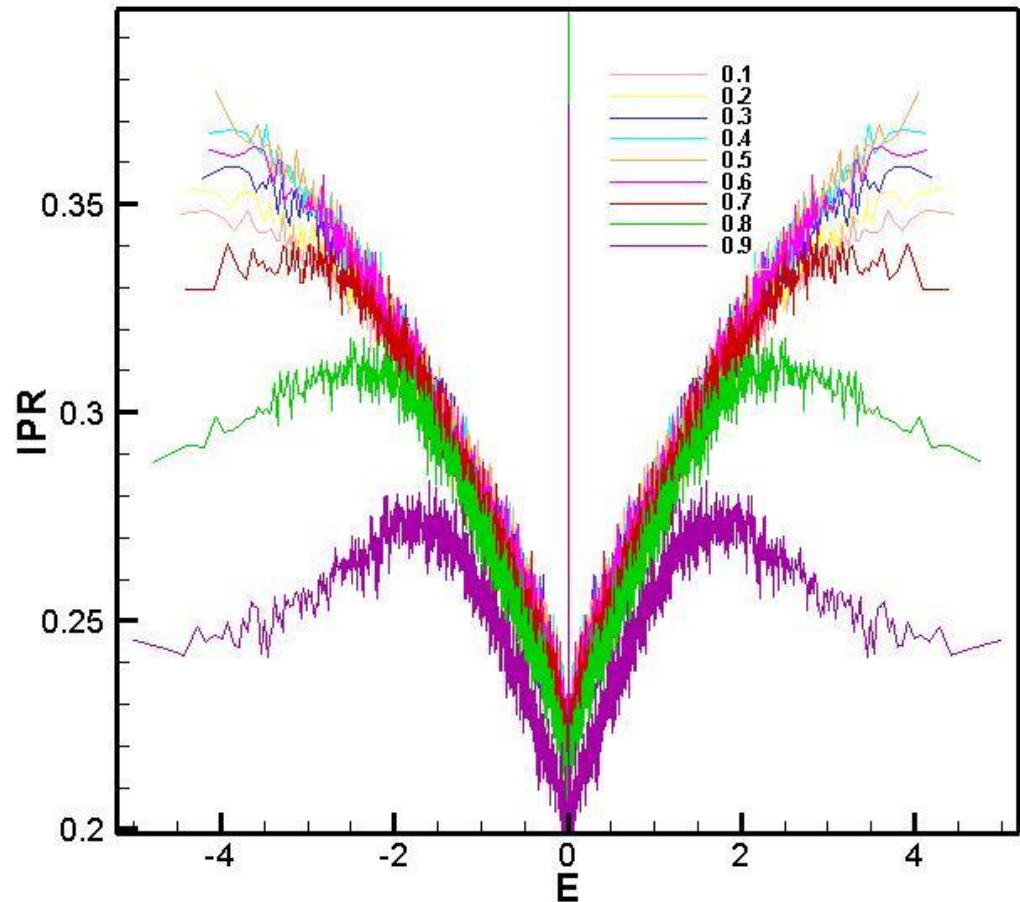
In upper energies,
upper exponents,
illustrate slump.



$N=2000$ by $H=0.1:0.1:0.9$
which was averaged by 500 samples.

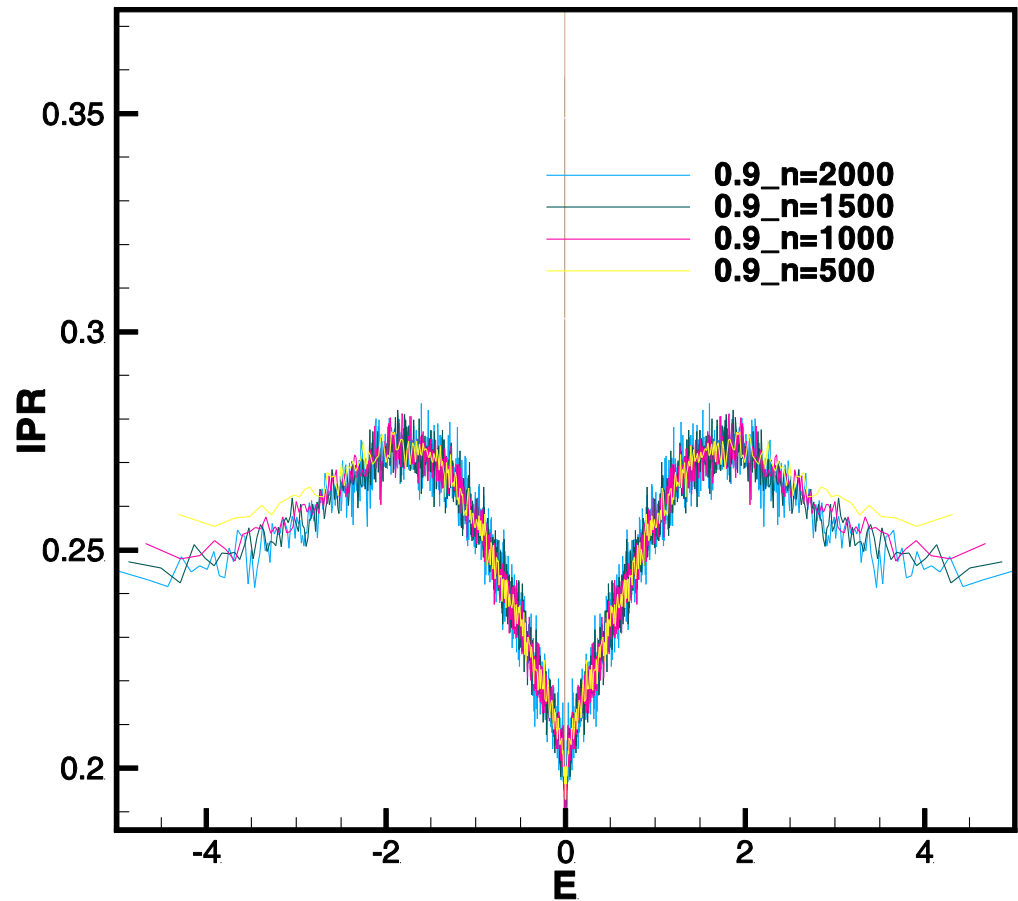
IPR's behavior
Changes with
different exponents.

Results illuminate
that for Upper H ,
the behavior
is not univocal.



N=various numbers
H=0.9

This diagram illustrates that the behavior of the system is not dependent on the sizes.
(in our case with our samples)



Results

Various disorders due to different exponents, belong to the same (unique) class of random distribution (Gaussian ensembles).

By changing the correlation between elements of random numbers through the changing of Hurst's exponents, we see different behaving and this changing is highlighted in noises with strong correlation ,(upper eigen value and lower eigen value has further effect on system.)

In the second step, we investigate the dynamic of spin lattice by using two dimensional fGn series for producing random coupling between various spins with Monte Carlo simulation.

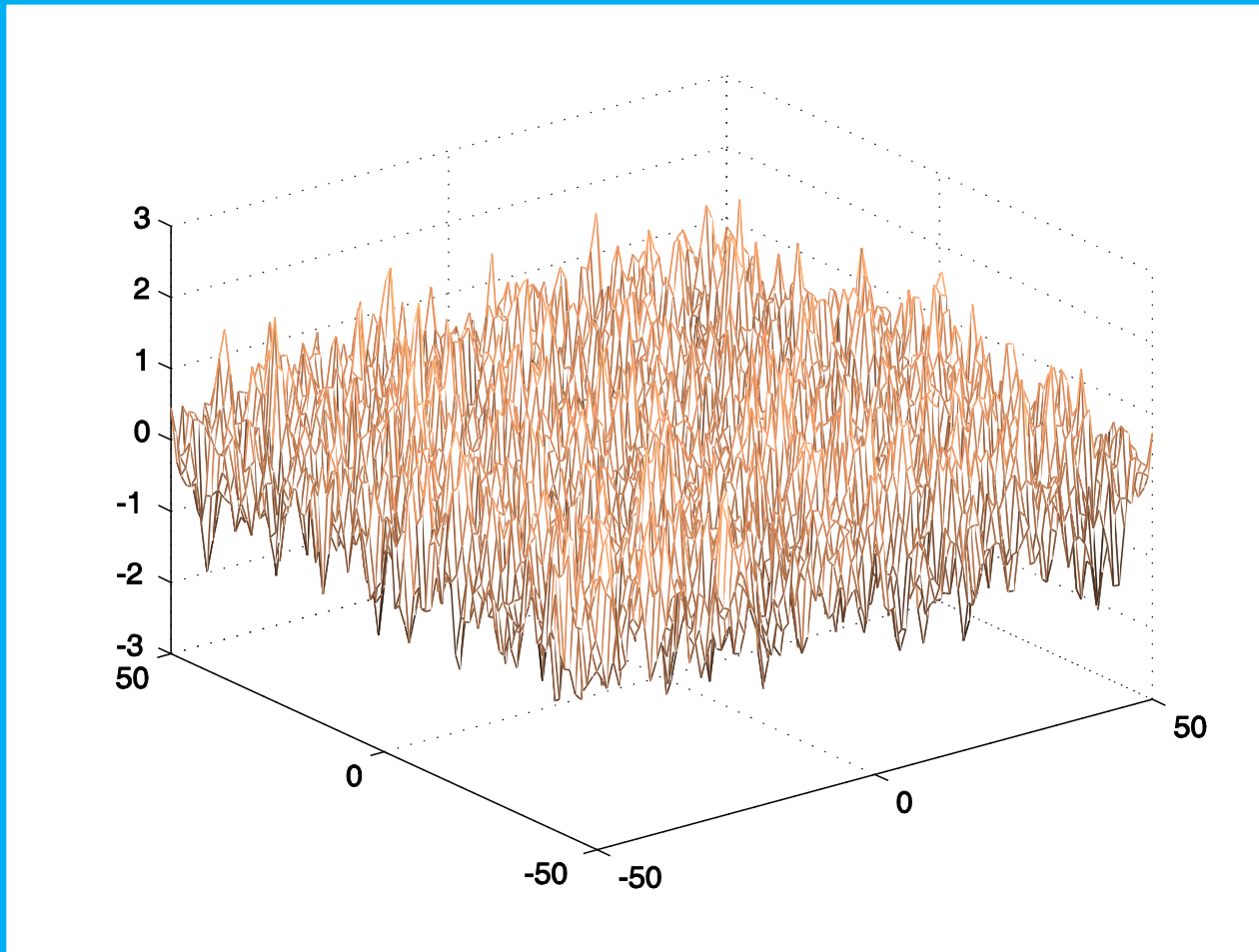
$$H = - \sum_{\langle ij \rangle} J_{i,j} S_i S_j$$

In one dimensional spin model, phase transition does **not appear**, because energy changes, even by maximum changes of spins (N/2 up & N/2 down) in a large N number of a system, is negligible so the distance between order or disorder ($\langle M \rangle = 0$ or $\langle M \rangle = 1$) is very small which cannot be seen.

So, for observing the dependency of phase transition point to those exponents, we studied the system in two dimension.

We did this simulation by Monte Carlo implementation for **different spin numbers**, **different irregularities**, **different coupling intensities** and **different temperatures** which the diagrams will be shown respectively and I will explain them.

Irregularities produced by two dimensional fGn

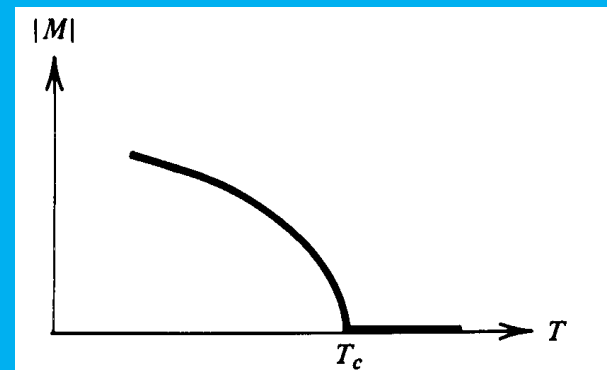


For the analysis of system behavior, we plotted magnetization versus temperature. Cause the magnetization is the order parameter of magnetic systems. And in the special temperature (critical) system shows a phase transition (ferromagnetic to para magnetic)

Magnetization is defined by m which is a function of temperature and external magnetic field.

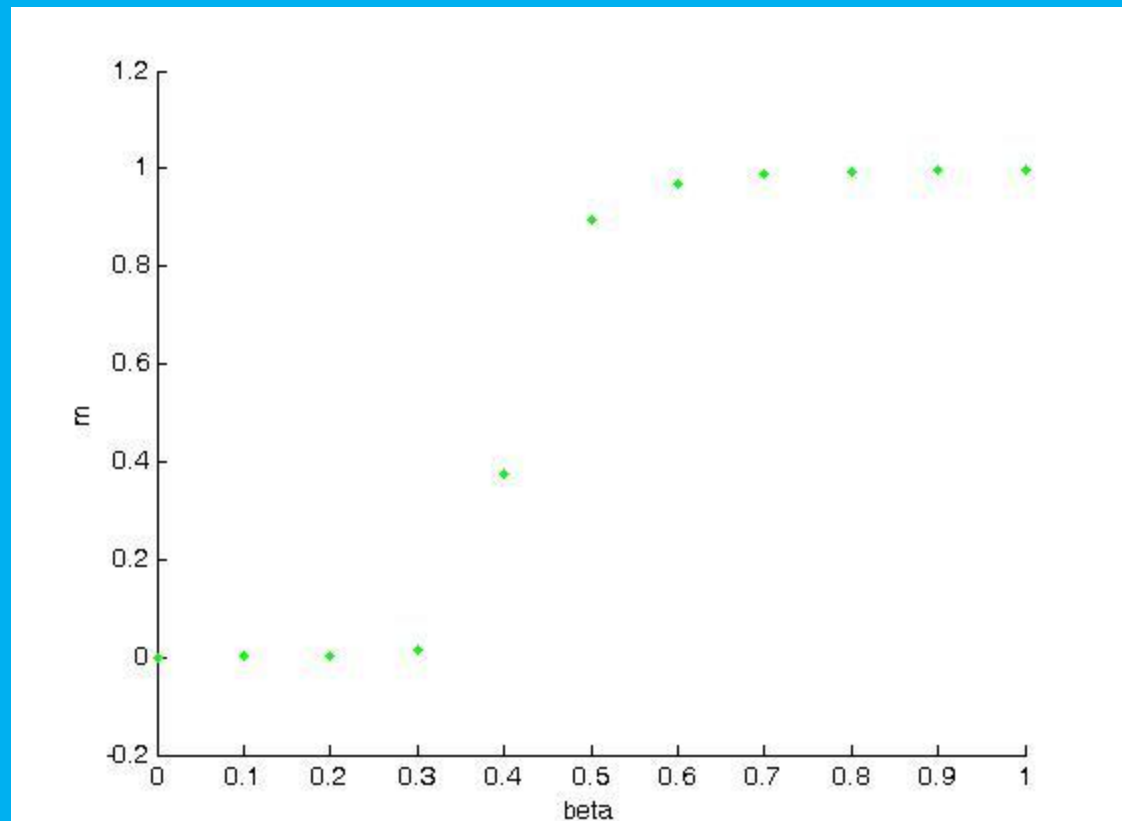
$$\mathbf{m} = \frac{1}{N} \sum_{i=1}^N (\mathbf{s}_i)$$

For the case $B=0$, there is a critical point which separates different phases. For example, system is ferromagnetic if $T_c > T$ and is para magnetic if $T_c < T$ and is unstationary if, $T_c = T$.



Magnetization curve versus beta(1/T) when we do not have any noise

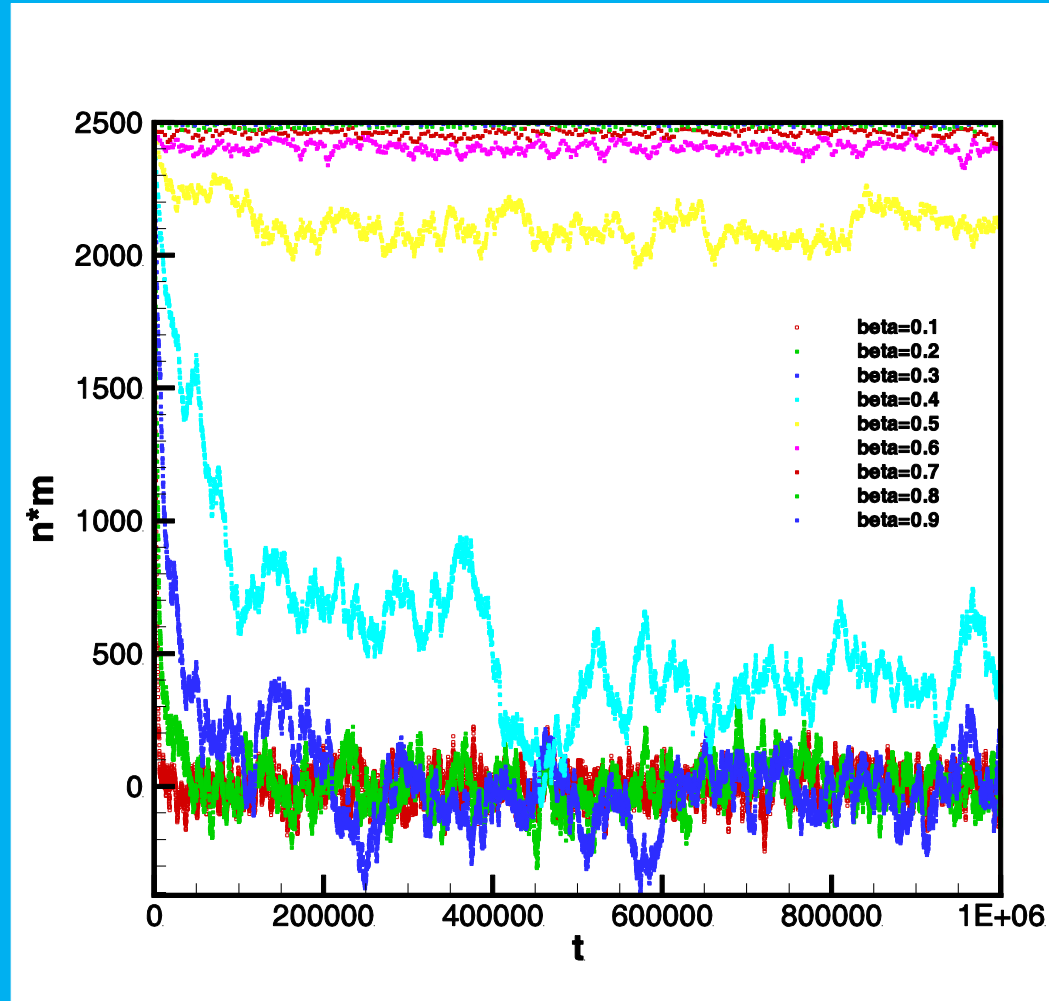
As we can see when the coupling coefficient is equal to a constant number, system illustrates a phase transition near $T = 2.269$ K($\beta = 0.4$) which is the transition between para magnetic and ferromagnetic.



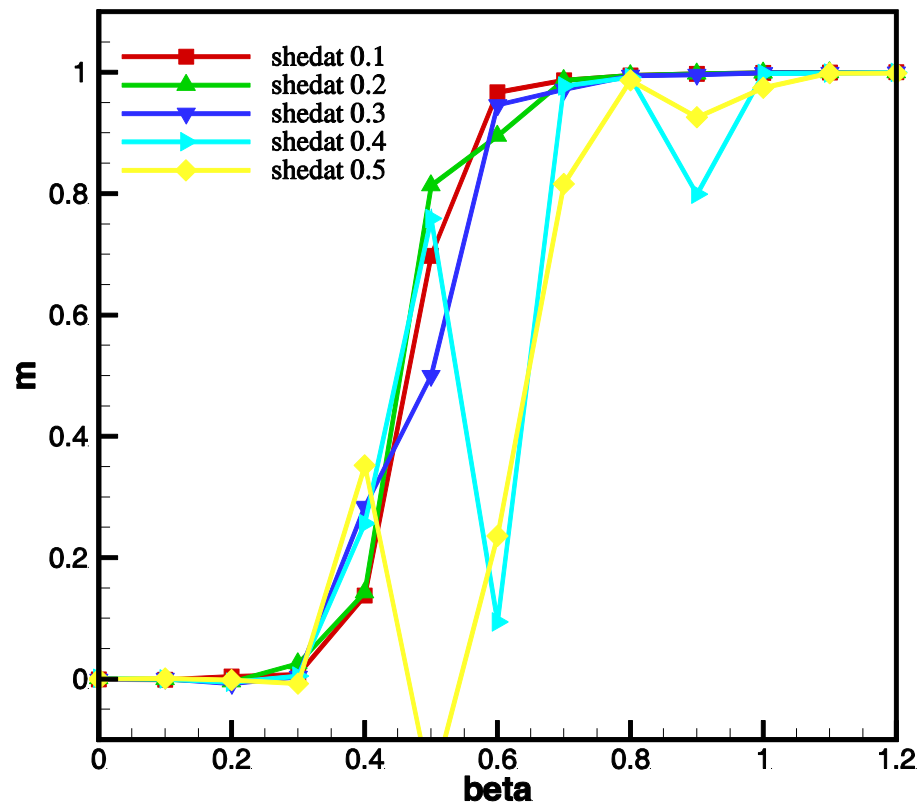
Magnetization curve versus time in different betas when we have no noise

In equilibrium, the magnetization reaches from 1 to zero.

As we can see from $\beta=0.4$ a change appeared in the behavior of the magnetization and it is an exponential transition.



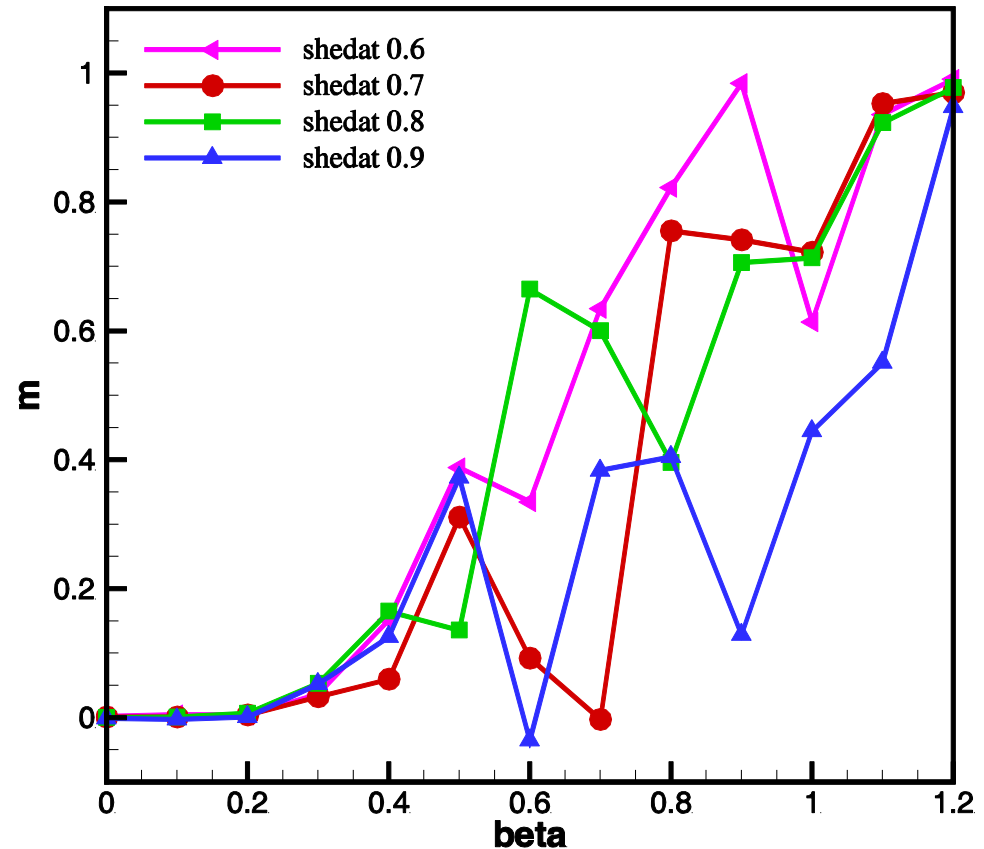
Now we add the noise to the system,
magnetization curve versus beta for various coupling
intensity (from 0.1 to 0.5) for different Hurst`s
exponential with N=50



For different coupling intensities (from 0.6 to 0.9)



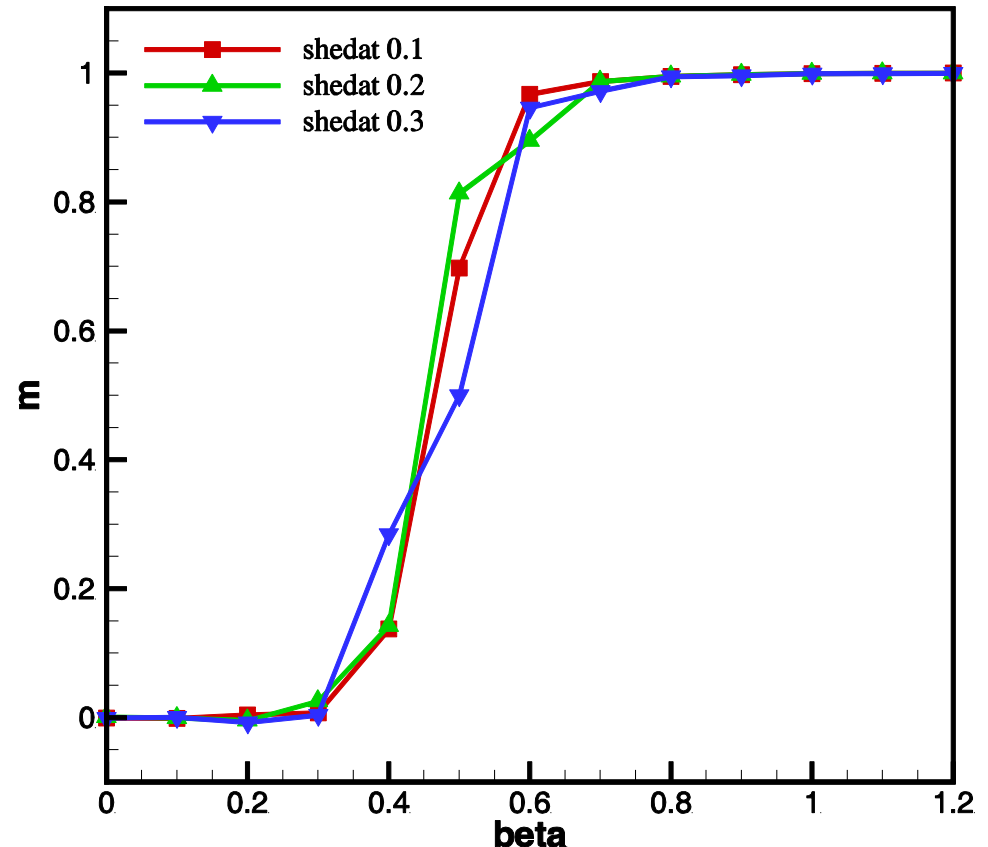
As it is clear
by increasing
the intensity of
irregularity,
the amount of
magnetization
fluctuation
become disordered
due to the temperature.



Magnetization curve versus beta for intensities (from 0.1 to 0.3) for different Hurts exponential ($H=0.1:0.1:0.9$) with $N=50$



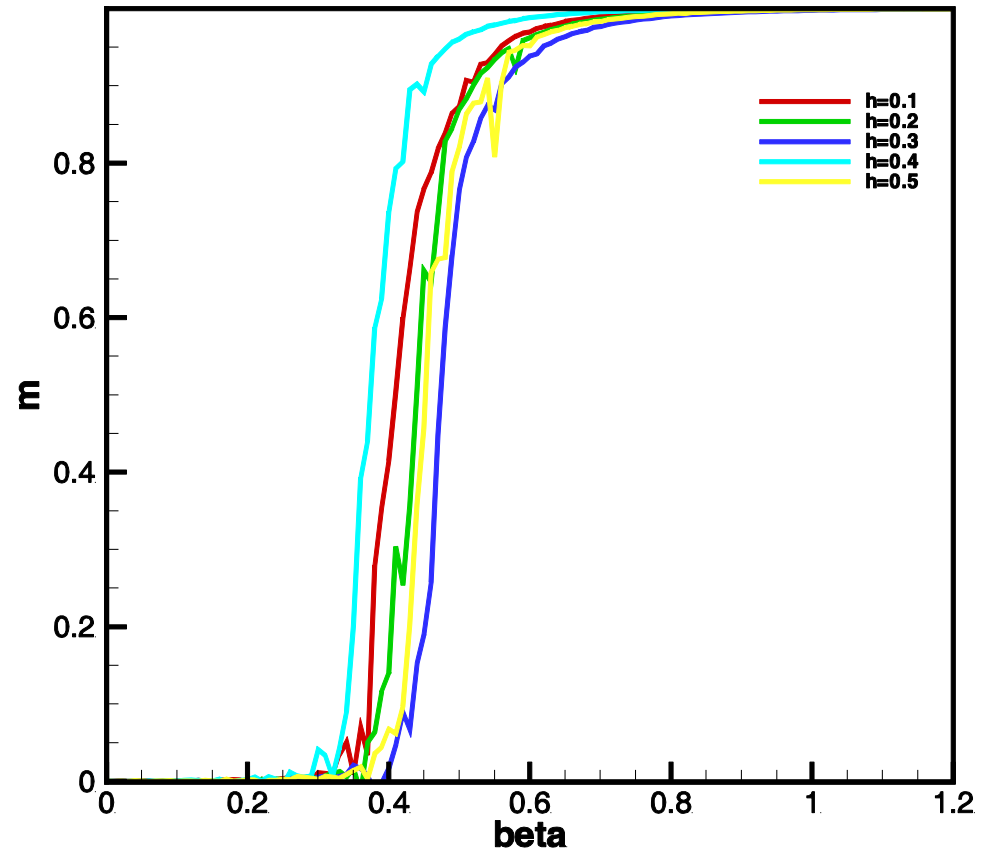
By discussing different intensities we reach to this result that the 0.2 intensity which has softer curve is more near to our aim which is adding irregularities to the system up to where the phase transition diagram be recognizable for the system



Magnetization curve versus beta for intensity=0.2 by different Hurst's exponents (H=0.1:0.1:0.5 with N=100)



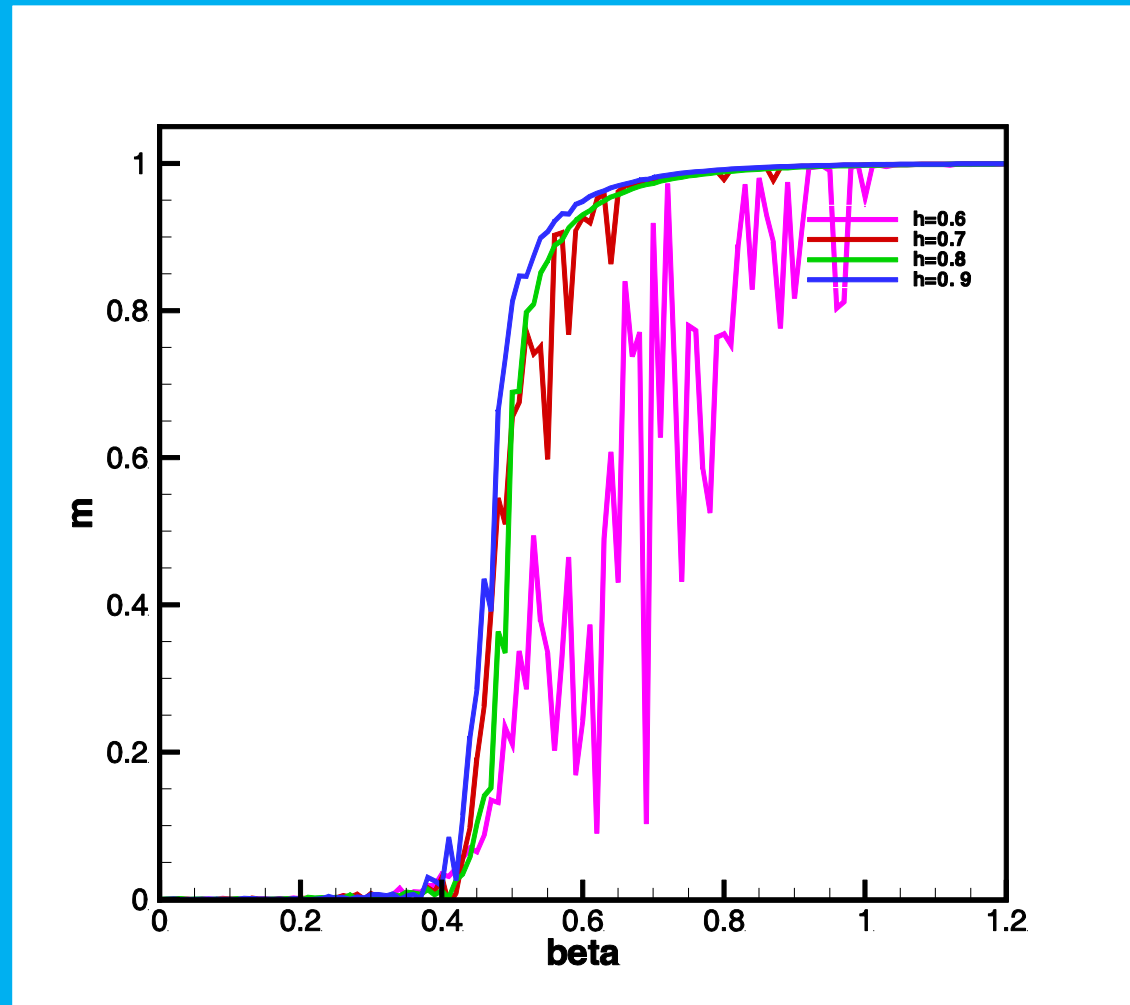
The phase transition behavior due to different exponents, has not specific order (forward or Backwad)



Magnetization curve versus beta for intensity=0.2 by different Hurst's exponents($H=0.6:0.1:0.9$) with $N=100$

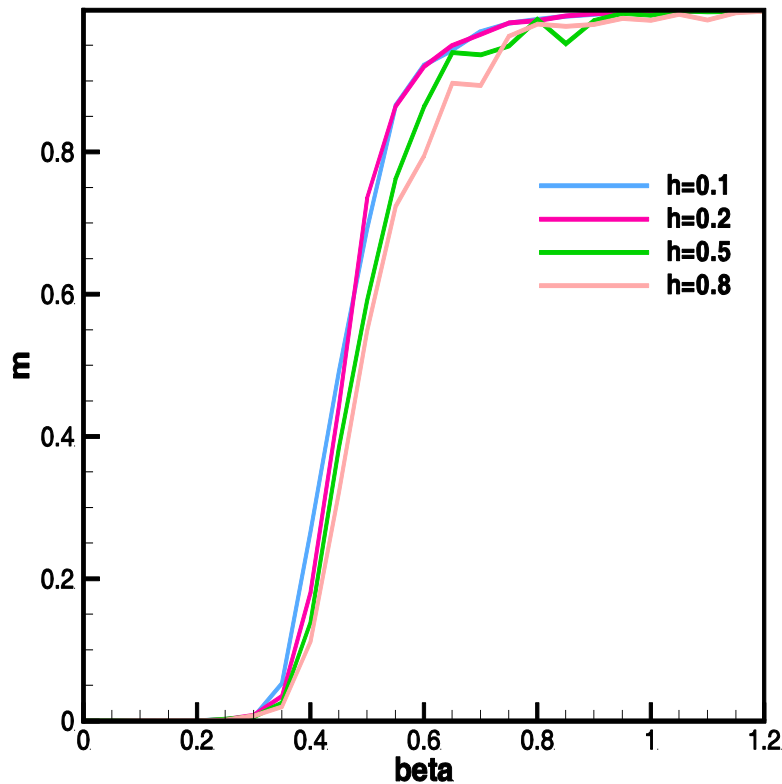
As it is evident, by changing the exponents, specific behavior has not seen.

So we can guess the transition point of the system is related to changing the configuration of the irregularities and the configuration changes have more impact than the changes of exponents.

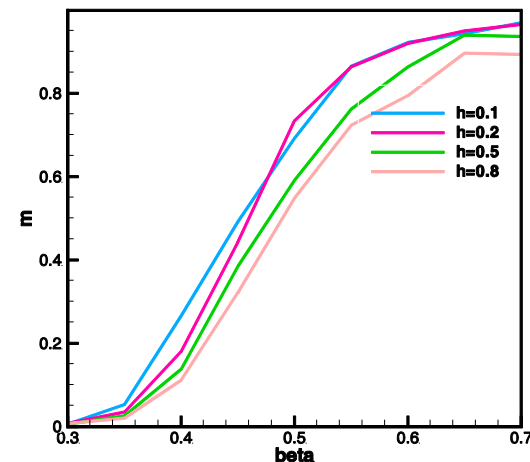


So averaging over different configuration is required that we did it for 4 exponents.

Averaged over 100samples.



It is clear that the lower exponent has better distribution in compare with upper exponent and lower exponent include more ferromagnetic spaces.



So ensembling indicates that results were dependent to the ensembles and by different ensembling and averaging over them, we can get a better information.

To study the susceptibility ,we plotted magnetization versus external field

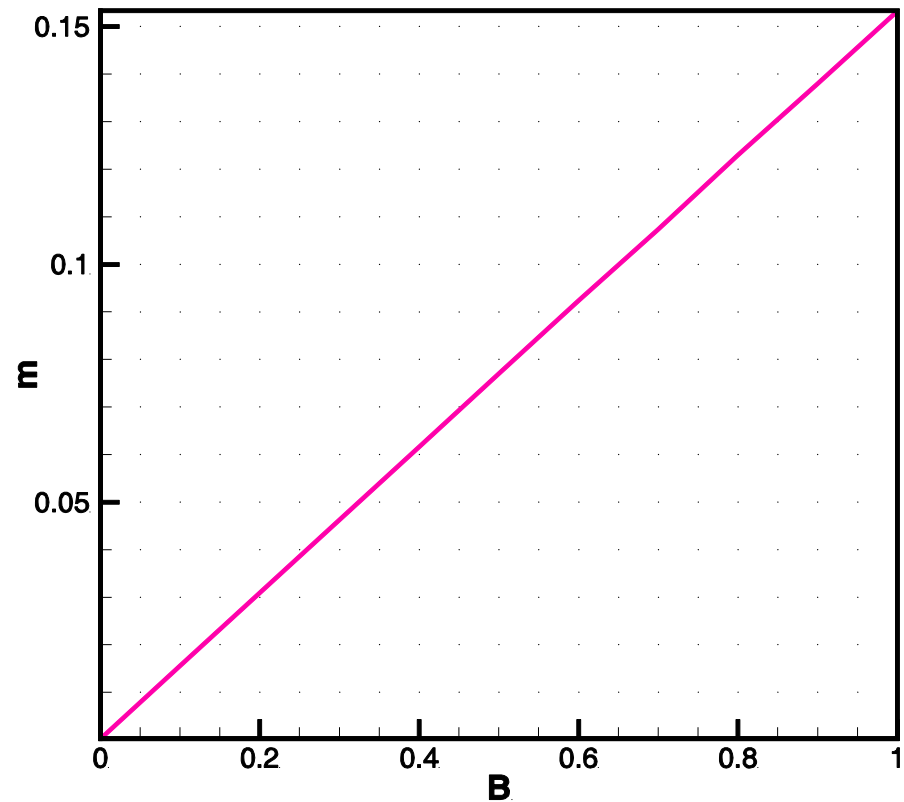
$N=40$

$B=0.0:0.1:1.0$

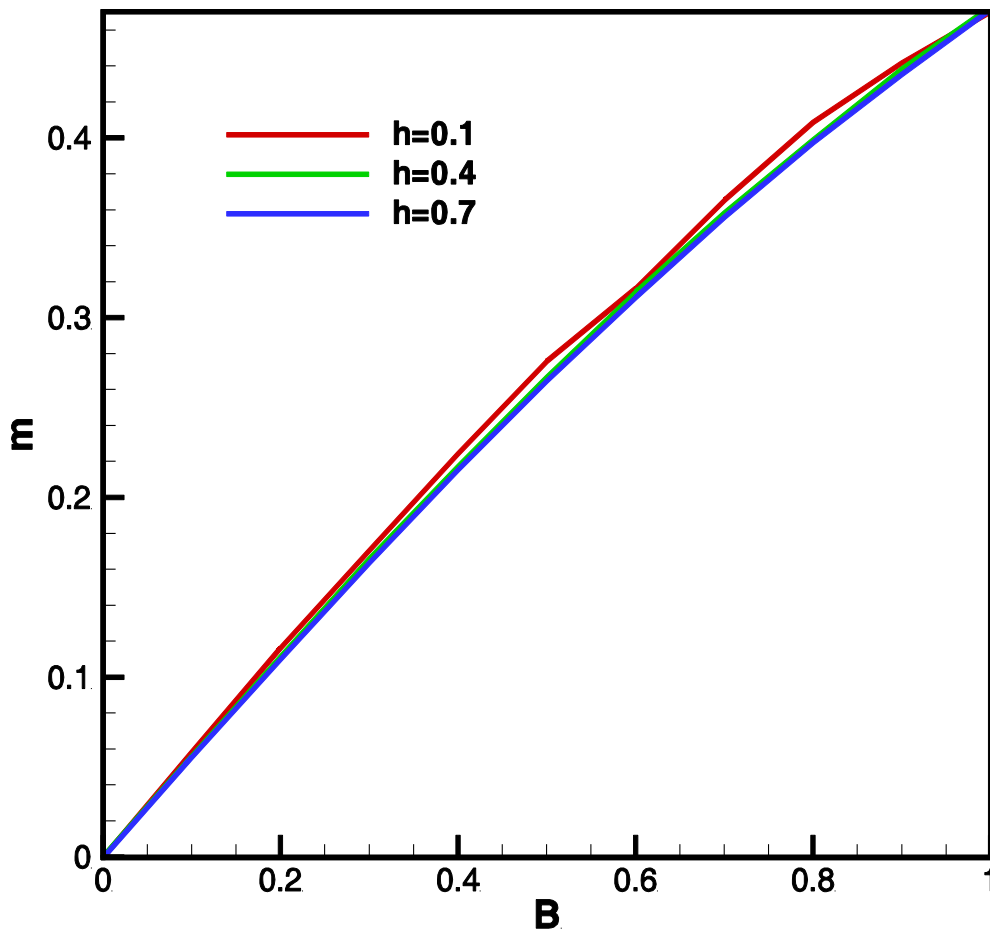
$\beta=0.2$

By increasing the magnetic field, magnetization increases linearly.

$X=m/B=0.16$



Susceptibility in presence of noise



$N=40$

$\text{Beta}=0.2$

$B=0.0:0.1:1.0$

Ensembling by
100 samples

$J=0.2$

And three
exponents.

Magnetic
susceptibility
has been
increased.

$\chi=0.6$

Susceptibility in critical point

$N=40$

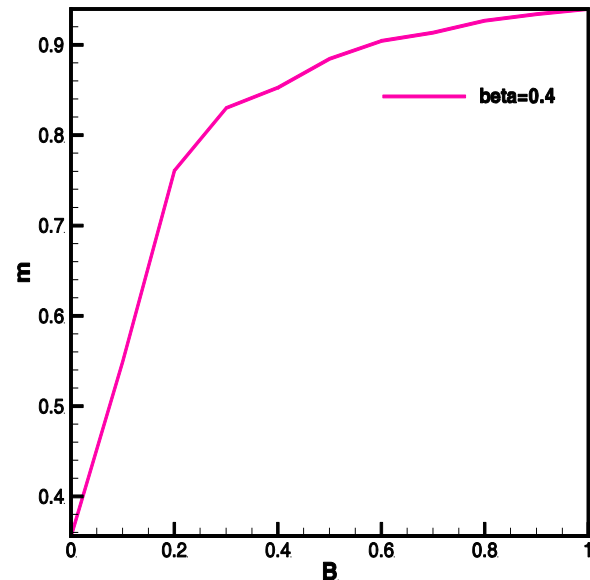
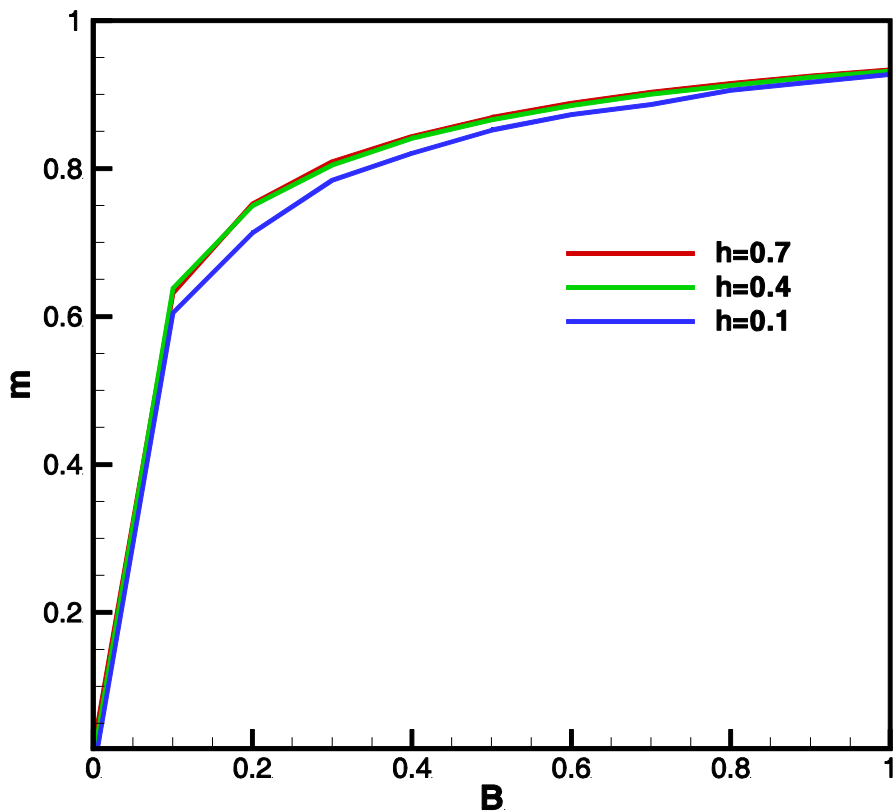
$\beta=0.4$

$B=0.0:0.1:1.0$

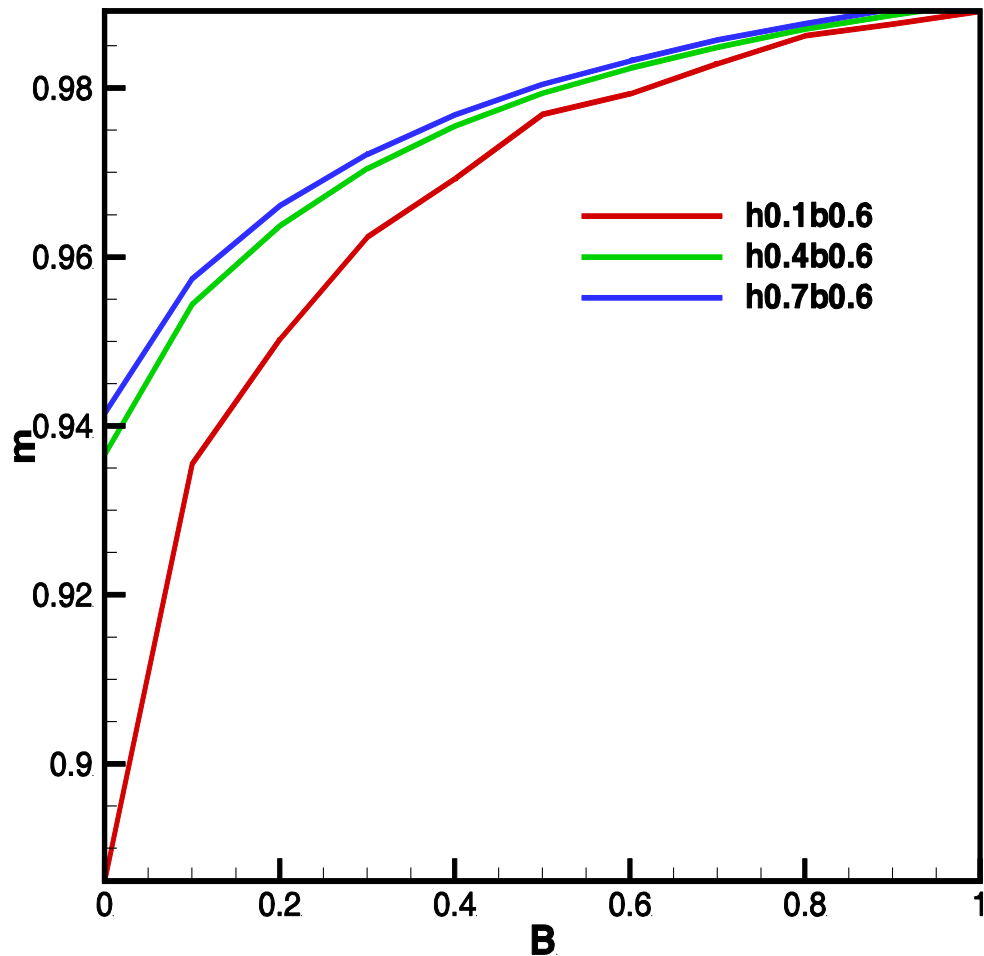
Ensembling by 100 samples

Magnetic susceptibility has been increased

$\chi=3.9 \rightarrow \chi=6$



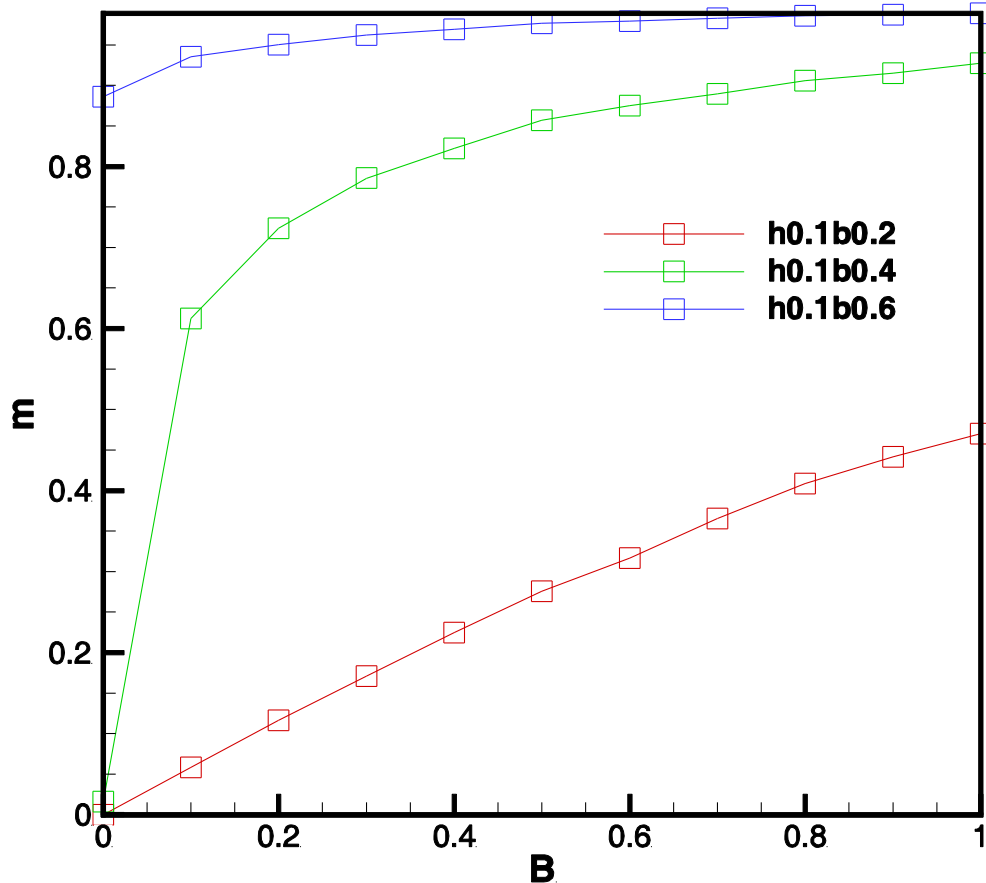
The effect of various irregularities



Beta=0.6

The
ferromagnetic
space for
different
exponents

Comparing susceptibility in 3 spaces



$N=40$

$\beta=0.2, 0.4, 0.6$

$B=0.0:0.1:1.0$

Ensembling by
100 samples

& one $H=0.1$

As we can see, the behavior of the susceptibility is different in three phases. In a specific exponent, increasing β (decreasing temperature), causes the increment of susceptibility.

Results

- As expected ,**phase transition** which is an important property of spin glass lattices , is observing by changing the irregularities .
- **Different set of irregularities** make a different result in a spin glass system and the impact of these configurations are more than the effect of different Hurts exponents of the fGn series.
- By **averaging** over the configurations ,the phase transition point will be dependent on the Hurts exponents as in smaller exponents, the space of ferromagnetic become more.
- By exerting irregularities, **susceptibility increases**.
- Various exponents illustrates different behavior in the ferromagnetic area while it shows less in paramagnetic area and transition point.



Future works

- We can apply **several kind of magnetic** fields , and study the behavior of the system and through this work find a guide to discuss the real networks like biological systems and the economic ones.
- Also we can use from **other random series** as a noise instead of fGn series for coupling the spins therefore using those data to forecast the behavior of some disordered systems.

The end

